Advances in Modeling Dielectric Breakdown

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This work was supported in part by AFOSR (Counter High Power Microwave Consortium, and Cathodes and Breakdown MURI), and DOE (Insulator Breakdown STTR). Key researchers include Dr. S. K. Nam, Dr. H. C. Kim, M. Aldan, and S. Taverniers.
Single-Surface RF Multipactor

- Multipactor discharge is a secondary electron avalanche frequently observed in microwave systems.

\[ E_y = E_{rf0} \sin(\omega t) \] leads to electron energy gain.

\[ \tau_{\text{transit}} = \frac{2m v_{z,0}}{eE_{z0}} \] (life time)

\[ z_{\text{transit}} = \frac{m}{2} \frac{v_{z,0}^2}{eE_{z0}} \] (maximum distance)

\[ E_{iy} = f \left( \frac{E_{rf0}}{\omega}, \omega\tau_{\text{transit}}, \phi_0 = \omega t_0 \right) \]

Dielectric Window

Dominates at low pressure
Secondary Electron Model

- Energy and angular dependence of secondary emission coefficient

\[ \delta(E_i, \theta) = \delta_{max0} \left( 1 + \frac{k_s \delta^2}{2\pi} \right) \left\{ \begin{array}{ll}
(w e^{1-w})^k, & w \leq 3.6 \\
1.125, & w > 3.6 
\end{array} \right. \]

\[ w = \frac{E_i - E_0}{E_{max0}(1+k_s \theta^2/2\pi) - E_0} \]

\[ k = \begin{cases} 
0.56, & w < 1 \\
0.25, & 1 \leq w \leq 3.6 
\end{cases} \]

\[ E_i \] (Electron Impact Energy)

\[ \theta \] (Electron Impact Angle)

PIC Multipactor Susceptibility

**Discharge on**
(Positive growth rate)

- Include transverse variation of $E_{RF}$
- Absorb at transverse wall
- Neglect transverse space charge

**Discharge off**
low $\sigma$ due to
- Too high impact energy
- Too small impact energy

High field susceptibility becomes vertical in waveguide – no upper field cutoff
TE_{10} Multipactor Migration

At the beginning

At transient

Weak $E_{rf}$

Strong $E_{rf}$

Weak $E_{rf}$

At the steady state

$E_y^0 = 5 \text{ MV/m}$

2.85 GHz, Vacuum
Explanation of Migration

- Susceptibility Curve

At steady state

Discharge on
(Positive growth rate)

At transient

\[ E_{DC} \left[ \text{MV/m} \right] \times \left( \frac{1}{1\text{GHz}} \right)^{-1} \left( \frac{E_{\text{max}}}{400\text{eV}} \right)^{-1} \]

At steady state

Center

Periphery
Multipactoror Power

![Graphs showing power distribution over time](image)

- ~2% of the input EM power is absorbed
- The phase difference between the discharge power and input EM power means that the electrons are not totally in equilibrium with the local rf electric field.
Collisional Effects

- As the pressure increases, electron-impact ionization collisions dominate secondary electron emission as the electron source.
- At high pressures, the number of ions becomes comparable to that of electrons.

\[ E_{rf0} = 3 \text{MV/m at 1GHz, Argon} \]
PIC: Electron Mean Energy

- Electrons in the multipactor discharge gain their energy by being accelerated by the rf electric field during the transit time.

- At high pressures, electrons suffer many collisions and lose a significant amount of energy gained from the rf electric field.

\[ E_{rf0} = 2.82 \text{MV/m at 2.85GHz, Argon} \]
Density Profile at Transition (1 Torr)

- High-velocity electrons generated from the secondary emission can reach the volume discharge region.

\[ \tau_{\text{trans}} v_c \ll 1 \]
Secondary Yield vs. Pressure

- Below 10 Torr, the secondary yield is nearly unity so that secondary electron emission is sustained by itself.
- As the pressure increases and hence the volume discharge suppresses the secondary electron emission, it decreases to less than unity.

* For particles accumulated over a cycle
PIC: Electron Energy Distribution

- Below 50 Torr, the EEPF is bi-Maxwellian like
- At high pressures, the EEPF becomes cutoff, with the depletion of high-energy electrons $v_c > \omega$

$E_{rf_0} = 2.82\, \text{MV/m at 2.85GHz}, \text{Argon}$
**TE_{10} Mode Effects**

- Significant transverse loss of electrons to waveguide
- Lower growth rate
- Electron kinetic energy is not changed.

\[ E_y^0 = 1.41 \text{ MV/m} \]

2.85 GHz, 200 Torr
Breakdown Time: Gas Dependence

\[ \tau \text{ defined by } \frac{N(\tau)}{N(0)} = 10^8 \]

\[ E_{rf0} = 2.82 \text{MV/m at } 2.85\text{GHz} \]

\( \tau \text{ vs. } p \)

\( \tau \) (ns)

- Low pressure: \( \tau \) for xenon is the lowest, due to largest ionization frequency
- High pressure: \( \tau \) for neon is the lowest, because the total frequency of collisions leading to significant electron energy loss is lowest in neon
Scaling Law: Low $p$

Postulate: $n(t) = n_0 \exp(<v_t>t)$, with the mean ionization rate $<v_t> \sim v_{i,\text{max}}/2$

For $n(\tau)/n_0 = 10^8$, we obtain $\tau = 18.4/<v>$

At low $p$: $\frac{1}{\tau_f} \gg \omega \gg v_c$ with $\tau_f$ the electron flight time

Breakdown time: $\tau \sim \frac{1}{v_i} \sim \frac{1}{n_g \langle \sigma v \rangle} \sim \frac{1}{p}$

since $<\sigma v>$ changes slowly near $E \sim 500$ eV typical of low $p$ regime

Then the scaling law predicts:

$\tau(\text{Argon}) \sim 6.4 \text{ ns/p(Torr)}$,

$\tau(\text{Neon}) \sim 18 \text{ ns/p(Torr)}$,

$\tau(\text{Xenon}) \sim 2 \text{ ns/p(Torr)}$.
Scaling Law: High $p$

Ionization discharge regime, with $\omega \ll v$

Rate of change in electron KE:

$$\frac{dW}{dt} = \frac{e^2 E_0^2}{2m v_c} - \text{loss terms} \quad \text{with } E_0 \text{ the rf electric field}$$

Assume 50% of energy to loss terms, then

$$\tau = 6.8 \times 10^{-12} s \times \left( \frac{W_i}{10 eV} \right) \left( \frac{< \sigma v >_c}{10^{-13} \text{m}^3/\text{s}} \right) \left( \frac{p}{1 \text{ Torr}} \right) \left( \frac{1 \text{MV/m}}{E_{eff}} \right)^2$$

$$E_{eff} \equiv \frac{E_{rf0}}{\sqrt{2[1 + (\omega/v_c)^2]}}$$

Rearranging:

$$\frac{E_{eff}}{p} \left( \frac{V}{\text{cm} - \text{Torr}} \right) = \frac{0.026}{p \tau} \sqrt{\left( \frac{W_i}{10 eV} \right) \left( \frac{< \sigma v >_c}{10^{-13} \text{m}^3/\text{s}} \right)}$$

Scaling law predicts:

$$\frac{E_{eff}}{p} \left[ \frac{V}{(\text{cm} - \text{Torr})} \right] = \frac{0.064}{\sqrt{p \tau (\text{Torr} - \text{s})}}; \quad \text{Ar}$$

$$\frac{E_{eff}}{p} \left[ \frac{V}{(\text{cm} - \text{Torr})} \right] = \frac{0.045}{\sqrt{p \tau (\text{Torr} - \text{s})}}; \quad \text{Xe}$$

$$\frac{E_{eff}}{p} \left[ \frac{V}{(\text{cm} - \text{Torr})} \right] = \frac{0.037}{\sqrt{p \tau (\text{Torr} - \text{s})}}; \quad \text{Ne}$$

Breakdown Scaling Law

Low pressure regime:
surface multipactor dominated

\[ \tau \sim \frac{1}{v_i} \sim \frac{1}{n_g \langle \sigma v \rangle} \sim \frac{1}{p} \]

High pressure regime:
collision dominated \((v_c \gg \omega)\)
volumetric discharge

\[ \frac{E_{eff}}{p} \sim \frac{1}{\sqrt{p \tau}} \]
Enhanced Global Model* I

- Coupled continuity equations for all species, e.g. for oxygen:

\[
\begin{align*}
\frac{dn_e}{dt} &= K_{ion} n_e n_{gas} - K_{att} n_e n_{gas} - K_{rec} n_e n_{O_2^+} + K_{det} n_e n_{O^-} + K_{det2} n_{O_2^+} n_{O^-} \\
\frac{dn_{O_2^+}}{dt} &= K_{ion} n_e n_{gas} - K_{rec} n_e n_{O_2^+} - K_{mut} n_{O^-} n_{O_2^+} \\
\frac{dn_{O^-}}{dt} &= K_{att} n_e n_{gas} - K_{det} n_e n_{O^-} - K_{mut} n_{O^-} n_{O_2^+} - K_{det2} n_{O_2^+} n_{O^-}
\end{align*}
\]

where \( K = \int \sqrt{\frac{2e\varepsilon}{m_e}} \sigma(\varepsilon) f(\varepsilon) d\varepsilon \)

* Nam and Verboncoeur, APL 23, 231502 (2008)
Oxygen Reactions

1) \( e + O_2 \rightarrow e + O_2 \)

2) \( e + O_2 \rightarrow e + O_2(r) \)

3-6) \( e + O_2 \rightarrow e + O_2(\nu = n, n = 1, 2, 3, 4) \)

7) \( e + O_2 \rightarrow e + O_2(a^1\Delta_g) \)

8) \( e + O_2 \rightarrow e + O_2(b^1\Sigma_g^+) \)

9) \( e + O_2 \rightarrow O + O^- \)

10) \( e + O_2 \rightarrow e + O_2(c^1\Sigma_u^-, A^3\Sigma_u^+) \)

11) \( e + O_2 \rightarrow e + O(3P) + O(3P) \)

12) \( e + O_2 \rightarrow e + O(3P) + O(1D) \)

13) \( e + O_2 \rightarrow e + O(1D) + O(1D) \)

14) \( e + O_2 \rightarrow e + O_2^+ + e \)

15) \( e + O_2 \rightarrow e + O + O^*(3p^3P) \)

16) \( e + O_2^+ \rightarrow O + O \)

17) \( e + O^- \rightarrow e + O + e \)

18) \( O^- + O_2^+ \rightarrow O + O_2 \)

19) \( O^- + O_2 \rightarrow O + O_2 + e \)
Enhanced Global Model II

- Electron energy equation:

\[
\frac{d}{dt}\left(\frac{3}{2} n_e kT_{\text{eff}}\right) = P_{\text{abs}} - (\varepsilon_{\text{ion}} K_{\text{ion}} n_e n_{\text{gas}} + \sum_{\text{exc}} \varepsilon_{\text{exc}} K_{\text{exc}} n_e n_{\text{gas}} + \tilde{K}_{\text{mom}} n_e n_{\text{gas}})
\]

- Improved RF power absorption model:

\[
P_{\text{abs}} = \int \frac{e^2 n_e}{m \nu_m} \left( \frac{E_0}{\sqrt{2}} \frac{\nu_m}{\sqrt{\nu_m^2 + \omega^2}} \right)^2 f(\varepsilon) d\varepsilon
\]
Enhanced Global Model III

- The general EEDF equation in the isotropic velocity space *:
  \[ f(\epsilon) = c_1 \epsilon^{1/2} e^{-c_2 \epsilon^x} \]

- Maxwellian: \( x=1 \), Druyvestyn: \( x=2 \)

- Determine \( x \) by the ionization and dissociative attachment from PIC model:

\[
K_{ion} \bigg|_{PIC} - K_{att} \bigg|_{PIC} = \\
\int_{\epsilon_{ion}} \sqrt{\frac{2e\epsilon}{m_e}} \cdot \sigma_{ion}(\epsilon) \cdot c_1 \epsilon^{1/2} e^{-c \left( \frac{\epsilon}{T_{eff}} \right)^x} \ d\epsilon - \int_{\epsilon_{att}} \sqrt{\frac{2e\epsilon}{m_e}} \cdot \sigma_{att}(\epsilon) \cdot c_1 \epsilon^{1/2} e^{-c \left( \frac{\epsilon}{T_{eff}} \right)^x} \ d\epsilon
\]

Enhanced Global Model in Ar

Argon Gas
$E_0 = 2.82$ MV/m
$f = 2.85$ GHz

![Graph showing the relationship between pressure (Torr) and time ($\tau$) in ns for different models.]

- Black line: PIC/MC
- Blue line: GM (Maxwellian)
- Red line: GM (EEDF with $x = 6.5$)
EEP F in Argon

Argon Gas
$E_0 = 2.82 \, \text{MV/m}$
$f = 2.85 \, \text{GHz}$
$p = 760 \, \text{Torr}$
Breakdown time in oxygen

*Oxygen*

$E_0 = 4.23 \text{ MV/m}$

$f = 2.85 \text{ GHz}$

**PIC/MC**

**Global Model (GM)**
Breakdown Time in Air

Air
$E_0 = 4.23 \text{ MV/m}$

$f = 2.85 \text{ GHz}$

$f = 5.70 \text{ GHz}$
Energy loss rate and $T_{\text{eff}}$

$E_o = 4.23 \text{ MV/m}$

$f = 2.85 \text{ GHz}$
Applied E vs f for $K_{\text{ratio}} = 1$ in air

$K_{\text{ratio}} = \frac{K_{\text{att}}}{K_{\text{ion}}}$

![Graph showing the relationship between $E/p$ (V/cm/Torr) and $f/p$ (GHz/Torr) for different pressures (760 Torr, 500 Torr, 200 Torr, 100 Torr). The graph illustrates two regions: $K_{\text{ratio}} < 1$ (Breakdown Region) and $K_{\text{ratio}} > 1$ (Decay Region).]
Effect of Frequency

Constant \( x \) is inadequate at high frequency for Ramsauer gases

Oxygen
Argon
\( E_0 = 2.82 \text{ MV/m} \)
\( P = 500 \text{ Torr} \)
Effect of Frequency

**Argon**
\[ E_0 = 2.82 \text{ MV/m}, \quad p = 500 \text{ Torr} \]

**Xenon**
\[ E_0 = 2.82 \text{ MV/m}, \quad p = 700 \text{ Torr} \]

**Oxygen and Nitrogen**
\[ E_0 = 4.23 \text{ MV/m} \]
\[ p = 500 \text{ Torr} \]

Argon: frequency dependence of $x$

![Graph showing the frequency dependence of $x$](image-url)
Comparison to Experiment

Plasma Filamentary Arrays

- 1.5 MW, 140 GHz Gyrotron
- 3 shots with slow (B&W) and fast (color) cameras
- Filaments spaced slightly less than $\lambda/4$, propagate towards source
- Hypothesis: constructive interference of reflected/diffracted waves, propagation speed limited by diffusion of seed electrons

EM Wave Model

\[
\frac{\partial^2 Z}{\partial Z^2} + k_z^2 Z = 0
\]

\[
k_z^2 = k^2 - (k_{x,m}^2 + k_{y,n}^2)
\]

\[
E_\perp = \text{Re}(E_0 Z e^{-j\omega t})
\]

Z is spatial profile of \(E_\perp\)

vacuum (1 and 3):

\[
k_1^2 = k_3^2 = \frac{\omega}{c}
\]

plasma (2):

\[
k_2^2 = \frac{\omega}{c} \left(1 - \sum_i \frac{\omega_{p,i}^2(z)}{\omega(\omega + j \nu_{m,i}(z))} \right)^{1/2}
\]

\(\omega_{p,i}\) : plasma frequency

\(\nu_{m,i}\) : momentum transfer frequency

* H.C. Kim and J. P. Verboncoeur, Comp. Phys. Comm. 177 (2007) 118-121
Fluid Model

Particle Continuity and Electron Energy Equations

\[
\frac{\partial n_e}{\partial t} = -\nabla \cdot J_e + K_{ion} n_e n_{\text{gas}}, \quad J_e = -D_e \nabla n_e - \mu_e n_e E_{||}
\]

\[
E_{||} = \frac{D_i - D_e}{\mu_i + \mu_e} \frac{\nabla n_e}{n_e}
\]

\[
\frac{\partial}{\partial t} \left( \frac{3}{2} n_e T_e \right) = -\nabla \cdot q_e + P_{abs} - (\varepsilon_{ion} K_{ion} n_e n_{\text{gas}} + \varepsilon_{exc} K_{exc} n_e n_{\text{gas}} + \tilde{K}_{mom} n_e n_{\text{gas}})
\]

\[
q_e = -\frac{3}{2} D_e \nabla n_e T_e + \frac{5}{2} J_e T_e
\]

\[
P_{abs} = \frac{en_e}{m_e v_m} E_{\perp}^2 = \mu_e n_e E_{\perp}^2
\]

Filament Simulation Results

\[ E_0 = 5 \text{ MV/m}, f = 110 \text{ GHz}, p = 760 \text{ Torr} \quad t = 6670 \text{ T (T = one wave period)} \]

Increasing field strength decreases filament spacing as breakdown threshold is exceeded closer to the previous filament.
Conclusions

- Multipactor dominates at low $p$
  - No upper field cutoff in waveguide
- Multipactor and ionization discharge compete at intermediate pressure (10-50 Torr)
  - Mean energy decreases with pressure
  - Discharge moves from surface (microns) to volume as pressure increases
  - EEDF goes from bi-Maxwellian to highly cutoff with pressure
- Ionization discharge dominates at atmospheric pressure
- A general scaling law depending on the collision frequency is deduced
- Enhanced kinetic global model agrees with PIC model over 4 orders in pressure, 3 orders in $f$
  - EEDF shape nearly independent of $p$, $E$
  - EEDF shows some dependence on $f$ for Ramsauer gases
- Wave-fluid model reproduces filamentary experiment well
  - Filament distance slightly less than $\lambda/4$
  - Propagation speed $\sim$ ambipolar diffusion