Spatio-temporal instabilities in relativistic electron bunches

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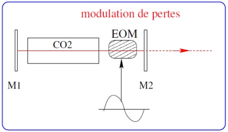
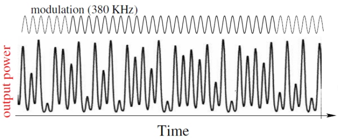
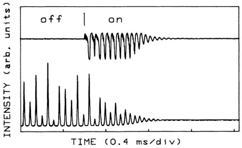
Nonlinear Optics/Nonlinear Dynamics group @ PhLAM

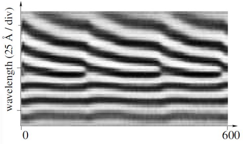
Mid 80s-90s

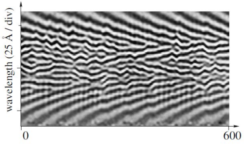
- Dynamics of lasers : experiments, modeling
- "Pure nonlinear dynamics", eg. chaos, control of chaos
- With space and time : shift from problems involving ordinary differential equations to problems involving partial differential equations











Nonlinear Optics/Nonlinear Dynamics group @ PhLAM

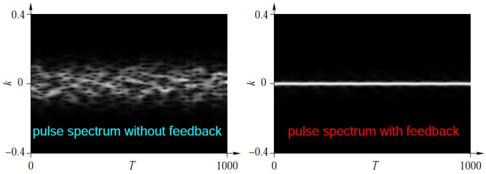
From 2000 (schematically)

- New activities in optics : propagation in photonic fibers, short pulse lasers, etc.
- Shift to studies of non-linear dynamics and nonlinear optics in different fields : biology, cold atoms, free-electron lasers, accelerator physics

Activities in accelerator-related topics

Start : Free-Electron Lasers

- Suppression of instabilities in free-electron lasers : super-ACO, France (2003), UVSOR, Japan (2004)
- Non-linear dynamics of free-electron lasers (collaborations with UVSOR and SOLEIL)



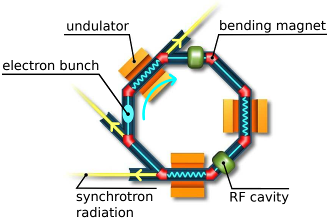
From ∼2006 : shift to dynamics of accelerators themselves

- Laser manipulation of electron bunches
- Instabilities of electron bunches in storage rings

Outline

1. Modeling
2. Spontaneous CSR
3. Laser seeding of CSR instability

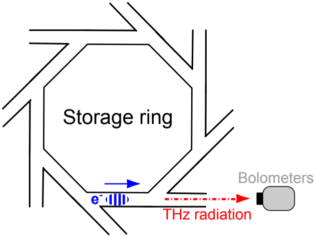
Synchrotron radiation in storage rings



- Electron bunch energy O(MeV-GeV)
- Electron bunch length O(mm-cm)
- Intense emission of synchrotron radiation
- Broadband : from far infrared to X-rays

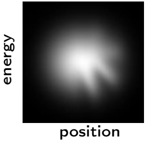
	Synchrotron SOLEIL (France)	UVSOR-III (Japan)
Beam energy (GeV)	2.75	0.6
Circumference (m)	354	53.2
Energy spread ($\Delta E/E$, rms)	1.017×10^{-3}	4.36×10^{-4}
Bunch length (mm, rms)	4.5	30

Microbunching instability (CSR instability) in storage rings



Bunch dynamics :

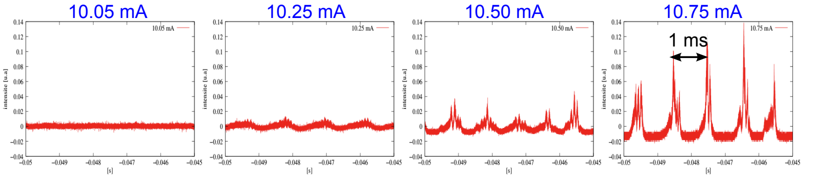
- If charge density > density threshold, interaction between the electron bunch and its radiation (wakefield) $\Rightarrow$ microbunching instability
- Formation of microstructures (at millimeter scale)



- Irregular evolutions in space and time

Experimental observations :

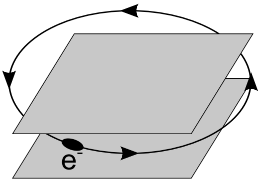
- Irregular emission of coherent synchrotron radiation (CSR) in the terahertz frequency domain



(Synchrotron SOLEIL, normal-alpha mode)

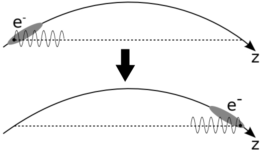
Longitudinal beam dynamics in storage rings

shielded CSR wakefield

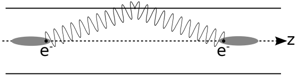


⇒

- Free space wakefield



- Parallel plates wakefield



The different ingredients use to model the dynamics are :

- the acceleration by the RF cavity,
- the radiation losses,
- the dependence of the round-trip time with electron energy,
- the interaction of the electrons with their own radiation (wakefields).

The Vlasov-Fokker-Planck (VFP) equation

Vlasov-Fokker-Planck (1D) equation

$$\frac{\partial f}{\partial \theta} = p\frac{\partial f}{\partial q} - q\frac{\partial f}{\partial p} \quad \Rightarrow \quad \text{rotation in phase space O(kHz)}$$

$$+ 2\varepsilon \frac{\partial}{\partial p}\left(pf + \frac{\partial f}{\partial p}\right) \quad \Rightarrow \quad \text{damping + diffusion O(ms)}$$

$$+ I_c E_{wf} \frac{\partial f}{\partial p} \quad \Rightarrow \quad \text{wakefield}$$

[Venturini and Warnock, Phys. Rev. Lett. 89, 224802 (2002)]

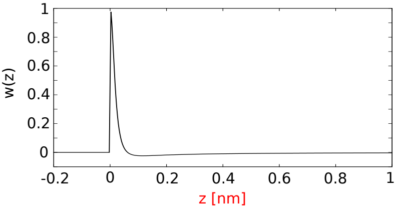
- $f(q, p, \theta)$: normalized electron distribution
- q : longitudinal position (in units of r.m.s. bunch length at equilibrium)
- p : relative energy (in units of relative energy spread at equilibrium)
- θ : time (dimensionless, 2π = one synchrotron period)

- $E_{wf}(q)$: electron moving on a circular orbit in the midplane between two parallel plates of infinite conductivity.
  [Murphy et al, Part. Acc. (1997)]

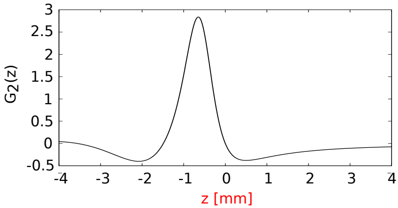
Wakefield computation

$$E_{wf}(q) = \frac{4}{3}\frac{\gamma^4 T_0}{4\pi\epsilon_0 R^2}\int_{-\infty}^{+\infty} d\zeta \rho(q-\zeta) w\left(\frac{3\gamma^3}{2R}\sigma_z\zeta\right) - \frac{T_0}{8\pi\epsilon_0 h^2}\int_{-\infty}^{+\infty} d\zeta \rho(q-\zeta) G_2\left(\frac{\sigma_z}{2R(h/R)^{3/2}}\zeta\right)$$

- Free space wake function



- Parallel plates wake function



Convolution of the density ρ with the free space contribution w :

- Integration by parts : $\int_{-\infty}^{+\infty} d\zeta \rho'(q-\zeta) v\left(\frac{3\gamma^3}{2R}\sigma_z\zeta\right)$ with $w = v'$.
- Special preprocess of the function v : evaluation of the exact value of the integration of v near zero within a fine mesh step.

  [J. Qiang, Computer Physics Communications 181, 313 (2010)]

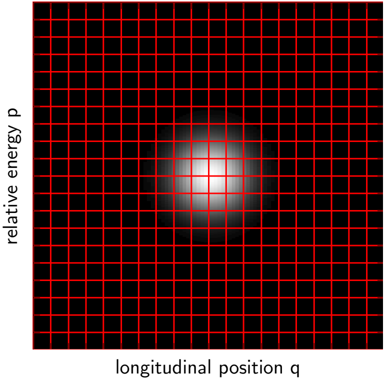
Integration method and numerical strategy

Semi-Lagrangian method[1] for the Vlasov part + Euler method for the Fokker-Planck term,
⇒ fast and well suited to parallelize with OpenMP and MPI.

1. Propagation over a small step $\Delta\theta$:

   $$f(q_i, p_j, \theta+\Delta\theta) = f(\mathcal{M}^{-1}(q_i, p_j), \theta)$$

   with $\mathcal{M}^{-1}$ the inverse map for the single particle motion
2. Nine point biquadratic interpolation to evaluate $f(q_i, p_j, \theta + \Delta\theta)$
3. Invoking operator splitting, propagation of f by the FP operator using Euler step.



1. [R. Warnock and J. Ellison, SLAC-PUB-8404, (2000)]

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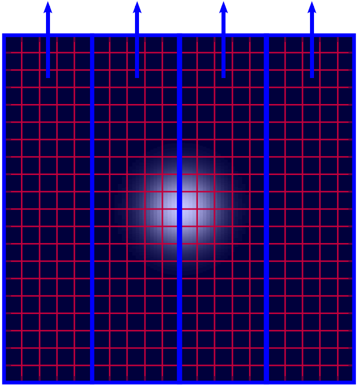

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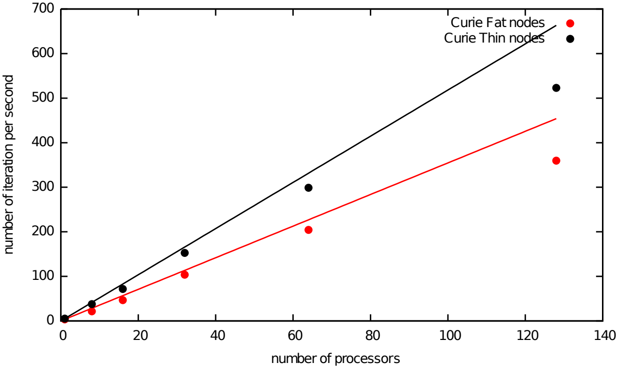
Parallelization on supercomputer

Test on the Curie supercomputer

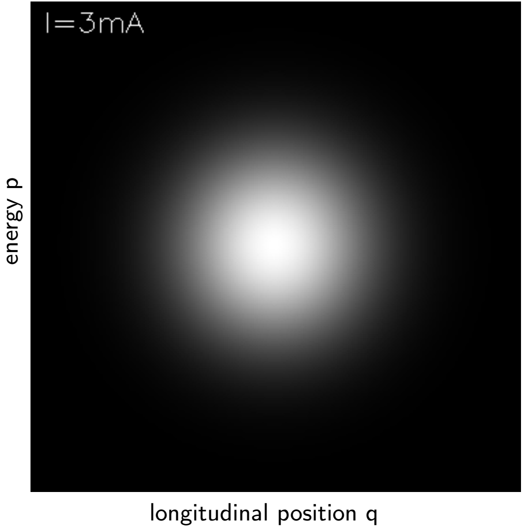


Typical integration duration (on lab's computer cluster) :

- **SOLEIL** : 2.1 minutes for 100 synchrotron periods transient on 32 CPU
- **UVSOR-II** : 42 minutes for 1000 synchrotron periods transient on 64 CPU

Longitudinal phase-space evolution

Integration of the VFP equation with the shielded CSR wakefield in the case of UVSOR-II in low-alpha mode.



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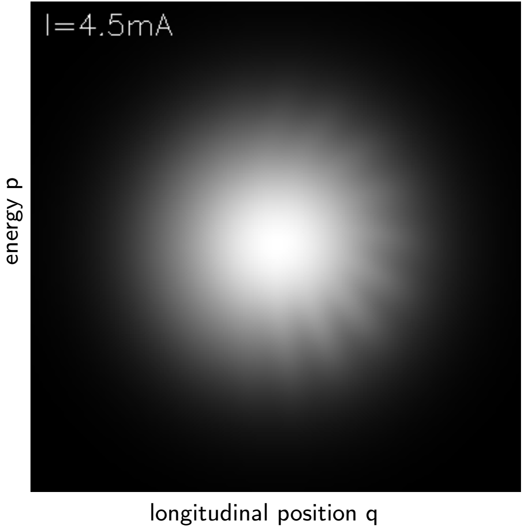
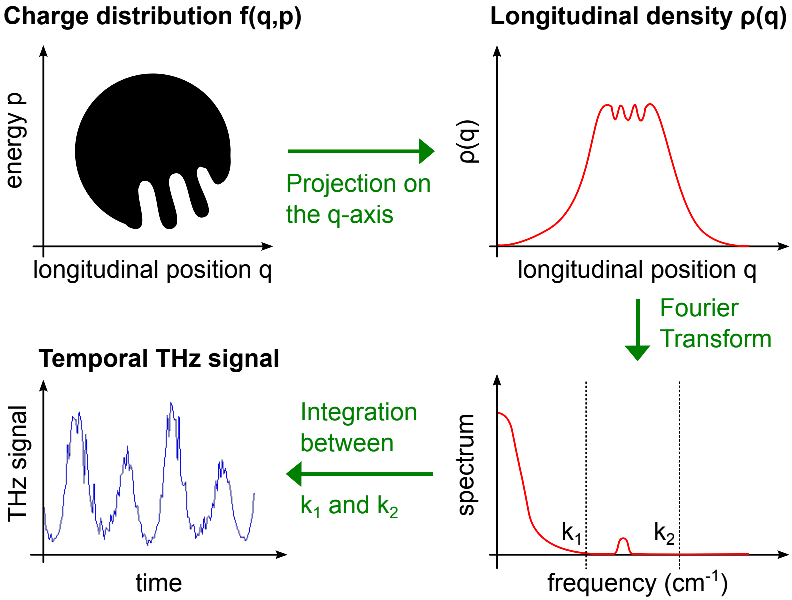
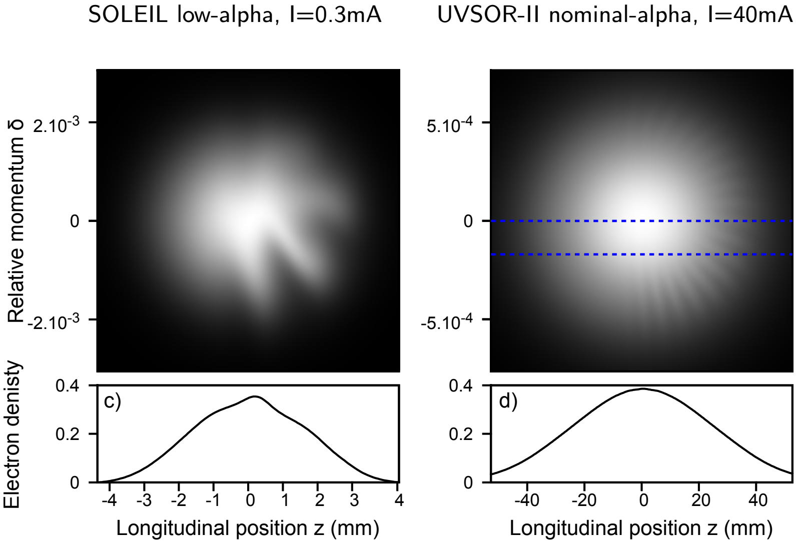


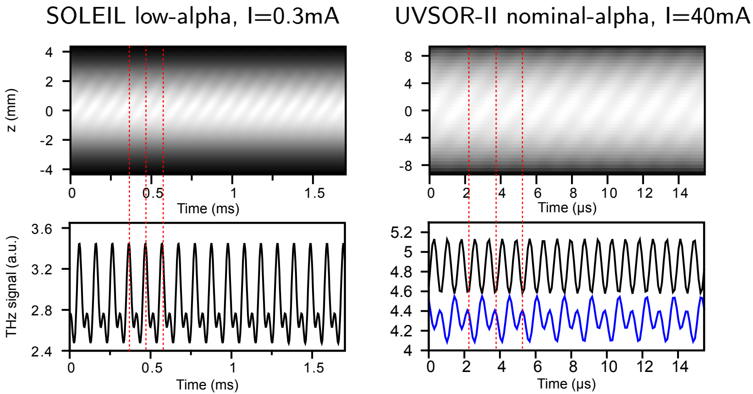
CSR : indirect measurement of the microstructures



Numerical results : longitudinal phase-space



Numerical results : THz signals



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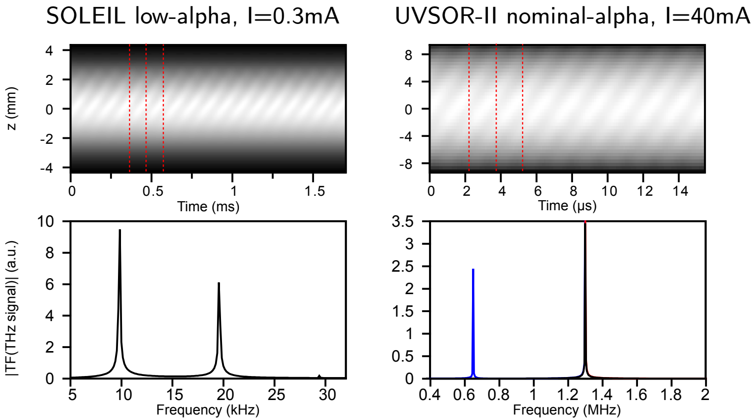
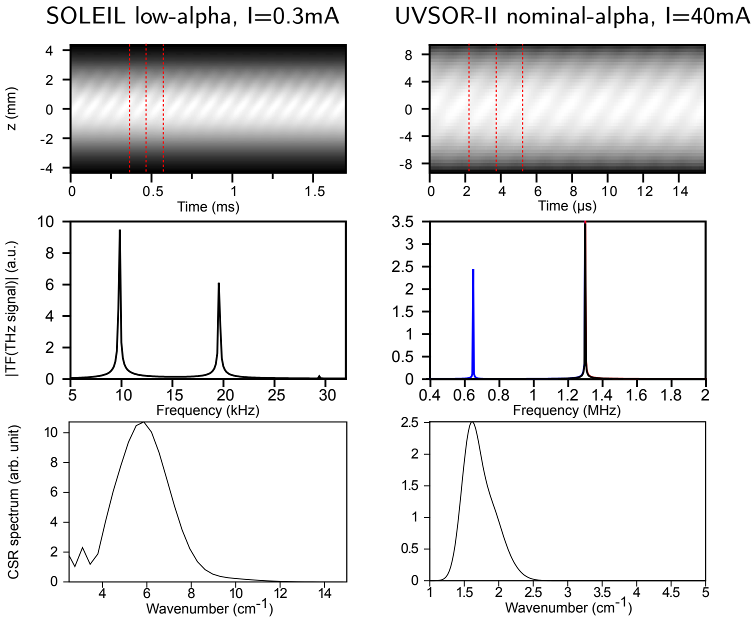
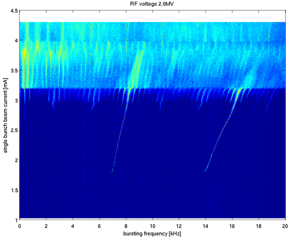


Numerical results : THz signals

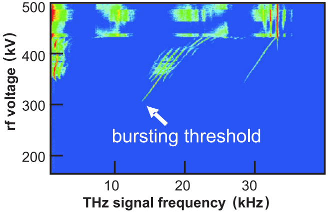


Experimental observations of the RF spectrum

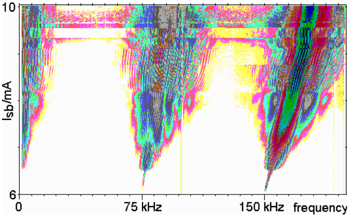
R. Bartolini et al., IPAC11, 3050 (2011)



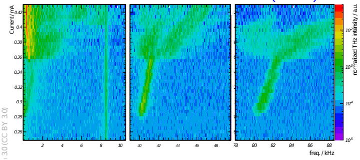
J. Feikes et al., Phys. Rev. ST Accel. Beams 14, 030705 (2011)



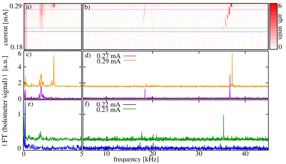
P. Kuske, PAC09, 4682 (2009)



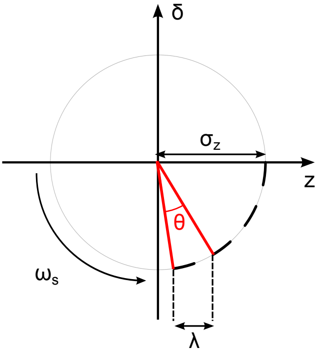
V. Judin et al., IPAC12, 1623 (2012)



C. Evain et al., EPL 98, 40006 (2012)



Link between the microstructure and the RF spectrum



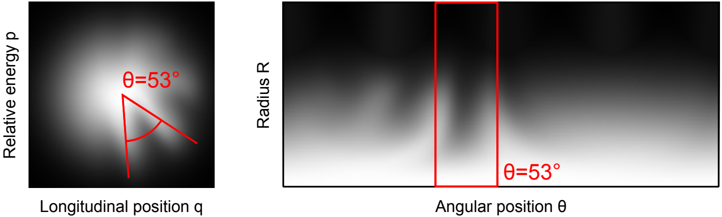
$$\theta = \frac{\omega_s}{f_m}$$

- f_m : temporal "fast" frequency of the THz signal
- ω_s : synchrotron pulsation
- θ : angle formed by 2 "fingers" of the microstructure

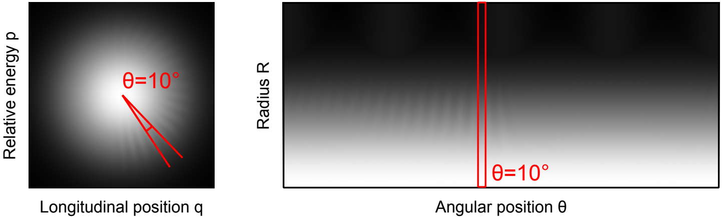
		SOLEIL	UVSOR
$f_s = \omega_s/2\pi$	(kHz)	1.47	19.4
f_m	(kHz)	10	650
θ	($^\circ$)	53	10

Polar representation of the phase-space

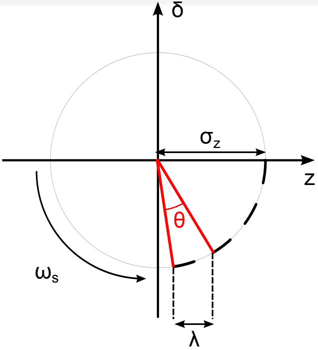
- SOLEIL low-alpha :



- UVSOR-II nominal alpha :



Estimation of the THz CSR wavenumber



$$f_m = \frac{\omega_s}{\theta} \approx \omega_s \frac{\sigma_z}{\lambda}$$

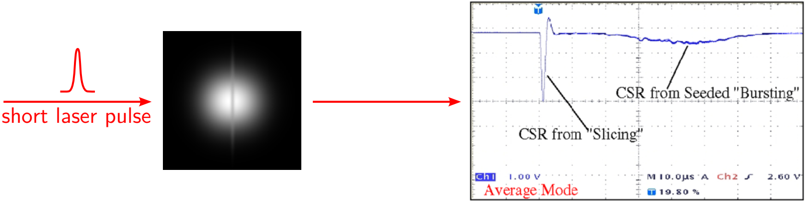
- f_m : temporal "fast" frequency of the THz signal
- ω_s : synchrotron pulsation
- σ_z : RMS bunch length
- λ : wavelength of the microstructure

		SOLEIL	UVSOR
$f_s = \omega_s/2\pi$	(kHz)	1.47	19.4
σ_z	(mm)	1.4	23.4
f_m	(kHz)	10	650
$1/\lambda_{formula}$	(cm^{-1})	7.7	2.3
$1/\lambda_{simulated}$	(cm^{-1})	6	1.7

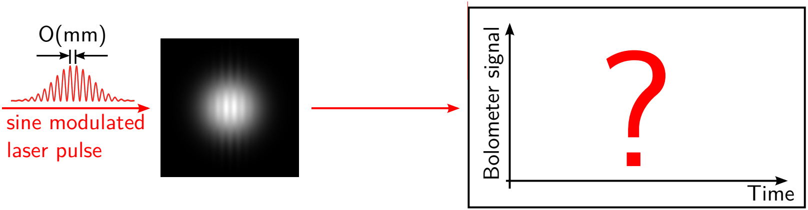
$\Rightarrow$ Frequency components in the RF spectrum of recorded THz signals may be related to the characteristic wavenumber of the microstructures in the bunch (and so to the wavenumber of the emitted signal).

Principle of seeding : initial bunching using an external laser

- Pioneering work : seeding in conditions of slicing
  [Byrd, Sannibale et al, Phys. Rev. Lett. 97, 074802 (2006)]



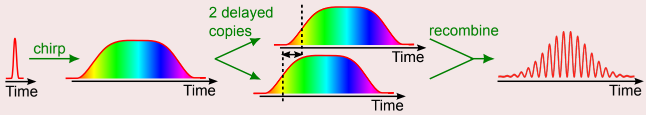
- Seeding with a modulated laser pulse ?



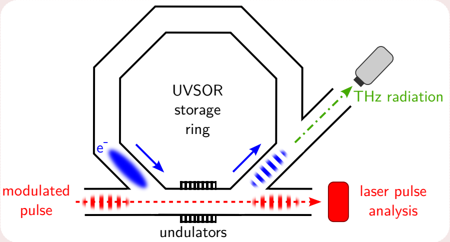
Experimental setup

Generation of modulated laser pulse : Chirped pulse beating

[Weling and Auston, JOSA B 13, 2783 (1996)]



Global setup



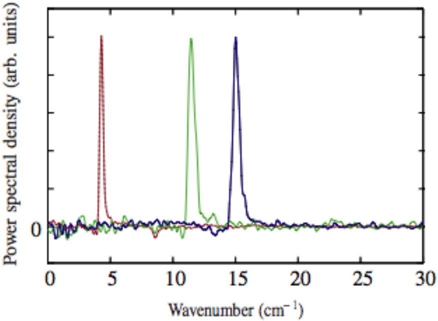
- UVSOR-II, normal and low-alpha and single bunch mode.
- Energy 600 Mev, relative energy spread $\approx 3.4 \times 10^{-4}$ and rms bunch lenght $\approx$ 3 cm.

Tests at very low beam current : effective bunching at mm scale

Experiments at UVSOR :

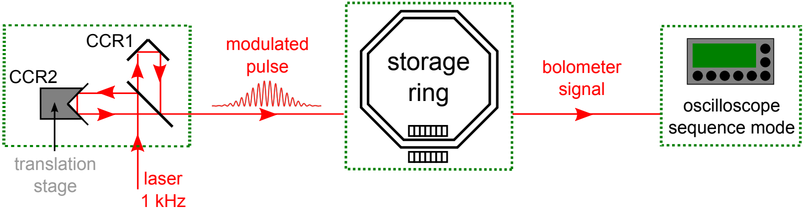
- [Hosaka et al, Phys. Rev. STAB 16, 020701 (2013)]
- [Evain et al, Phys. Rev. STAB 13, 090703 (2010)]
- [Bielawski et al, Nature Phys. 4, 390 (2008)]

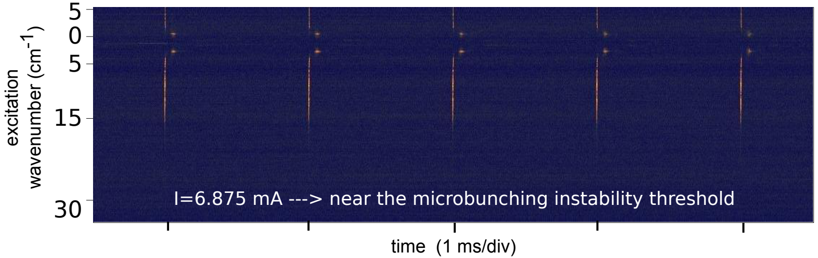
Typical emission spectra induced by the shaped laser



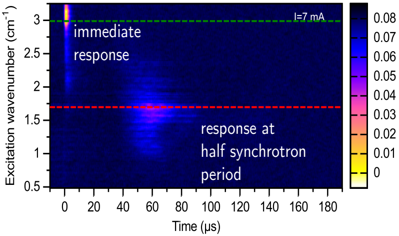
$\Rightarrow$ Observation of narrowband THz emission in a bending magnet

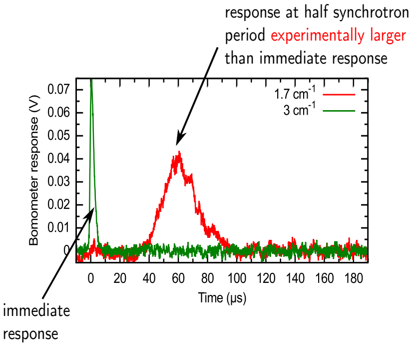
Seeding results at UVSOR-II in low-alpha mode

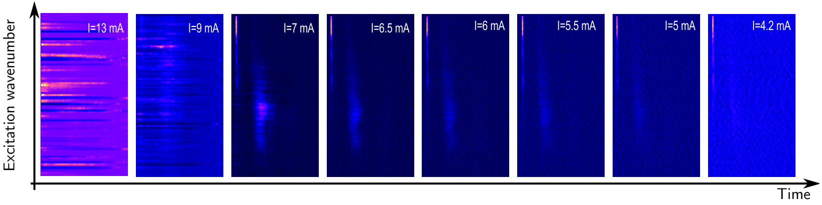




Zoom of bolometer signal versus excitation wavenumber

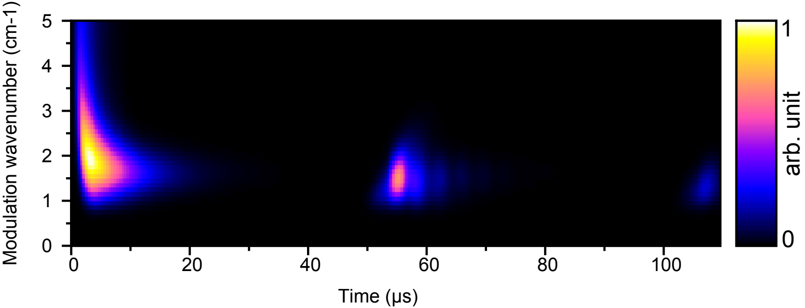






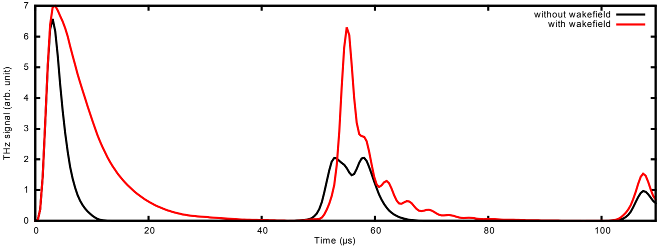
Numerical result : CSR versus time

For a beam current I = 3.5 mA (just below the microbunching instability threshold) :

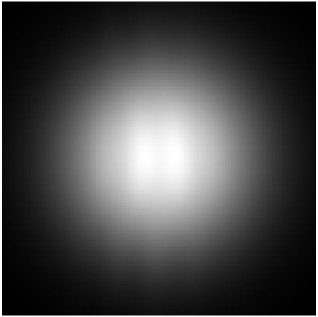


- Bolometer bandwidth : 1 to 9.1 cm^{-1}
- Bolometer time response : 2 μs

Typical simulated signal for an excitation at 1.5 cm^{-1}



without wakefield :



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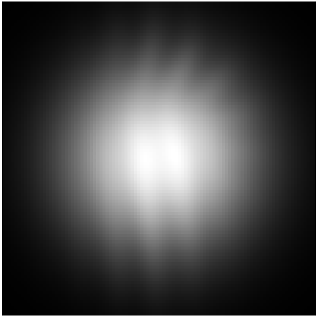
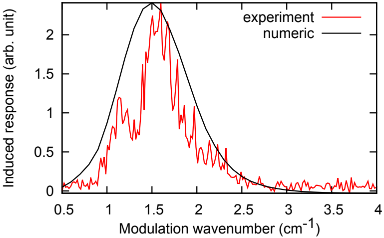


energy p

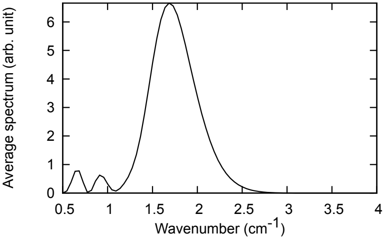
longitudinal position q

Resonance curve

- Maximum value of the delayed response



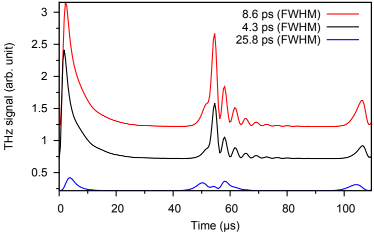
- Numerical average spectrum of the spontaneous CSR

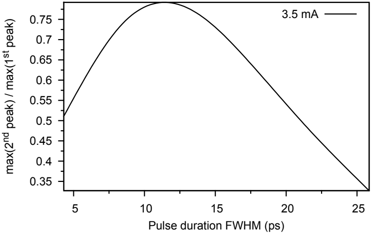


- Experimental and numerical resonance wavenumber : 1.6 cm^{-1}
- Response at half synchrotron period : resonance curve at the characteristic wavenumbers of the system.

Numerical calculation of the response to a short laser pulse

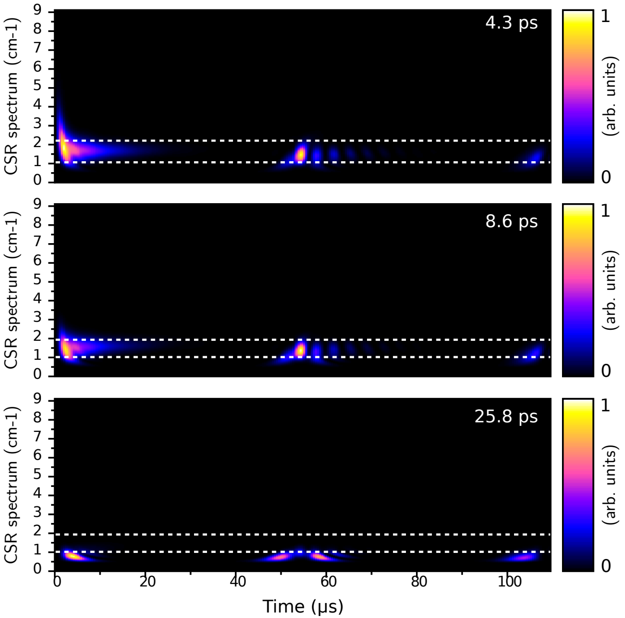
THz signal for a beam current of 3.5 mA with various pulse durations :





$\Rightarrow$ Strong effect of the pulse duration on the delayed response in the THz signal. Here, maximum efficiency for a pulse duration of 10 ps. Why ?

Temporal CSR spectra



Conclusion

Modeling

- Numerical integration of the 1D VFP equation in parallel (OpenMP, MPI),
- Computation of the exact expression of the shielded CSR wakefield.

Spontaneous CSR

- Link between the RF frequency in the THz signal and the longitudinal phase-space microstructures, e.g. the wavelength of the microstructures.

Laser seeding

- Experimental seeding of the microbunching instability with modulated laser pulses,
- Good agreements with the simple 1D model (VFP + shielded CSR wakefield), e.g., the resonance wavenumber

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Wakefield computation

- Free-space contribution

  Integration by parts by computing the FFT of $v(\mu)$ (the antiderivative of $w(\mu)$) with a very fine mesh (typically 100 times finer than the mesh for $\rho(q)$) and multiply it by ik.
  [J. Qiang, Computer Physics Communications 181, 313 (2010)]

  1. Because near zero, fast variation (at a scale much shorter than one mesh step)
     $\rightarrow$ evaluation of the exact value of the integration of $v(\mu)$ near zero within a δq stepsize of the fine mesh
  2. Computation of the FFT of the array v_i :

$$
\begin{aligned}
v_0 &= \frac{1}{\delta q} \int_0^{\delta q/2} v\left(\frac{3\gamma^3}{2R}\sigma_z q\right) dq \\
v_i &= v\left(\frac{3\gamma^3}{2R}\sigma_z i\delta q\right) \text{ for } i > 0
\end{aligned}
$$

- Parallel-plates contribution

  Computation of the FFT of $G_2(x)$ using a very fine mesh.

Wakefield computation