Kinetic Models and Advanced Diagnostics for the Space-Charge Induced Microbunching Instability

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LBNL February 10th 2012

OUTLINE

- Introduction and Motivation: the Microbunching Instability in High Brightness Injectors
- 6-D Theory of Space-Charge Waves in Relativistic Electron beams: Eigenmode Analysis and Initial Value Problem
- Kinetic Theory of the Microbunching Instability and Noise suppression experiments.
- COTR Experiment for 3-D Reconstruction of Beam Microbunching

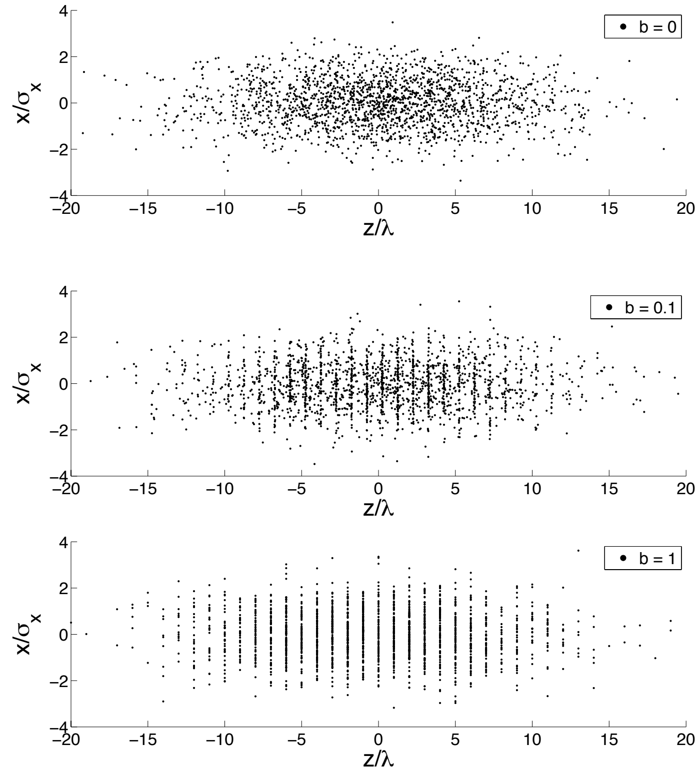
Fundamental Definitions: Beam Microbunching

$$b(k) = \frac{1}{N}\sum_n e^{-ikz_n}$$

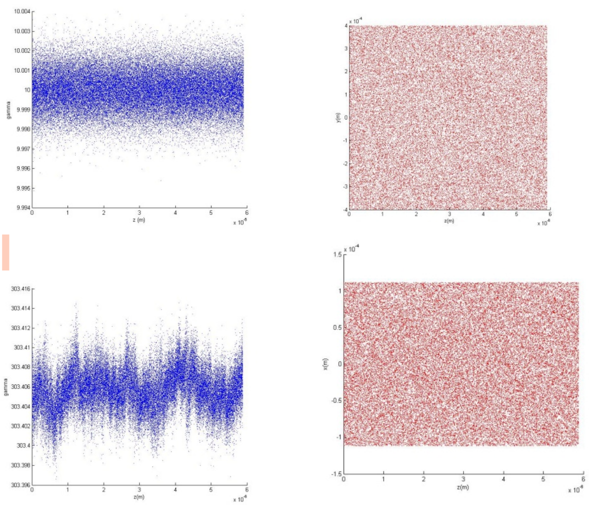
-Bunching Factor: Fourier transform of the beam current profile

-Shot-noise Microbunching: Random distribution of particles generates fluctuations in the current profile

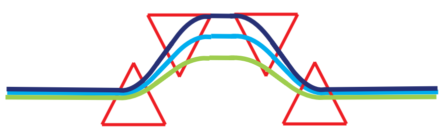
$$<|b(k)|^2> = \frac{1}{N^2}\sum_{m,n} e^{-ik(z_n - z_m)} = \frac{1}{N}$$

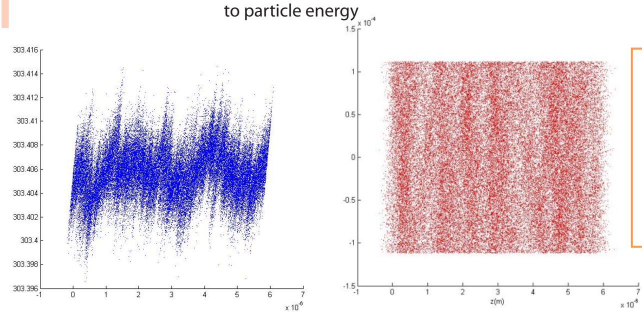


Motivation for This Work: Microbunching Instability in FEL Beams



Magnetic Chicane





Shot noise

↓

Fluctuations in longitudinal space-charge field

↓

Broad-band energy modulation

↓

Magnetic Chicane transfers modulation from energy to density

Emission of TR disturbs beam diagnostics!

Schmidt, et al., FEL 2009 (WEPC50)

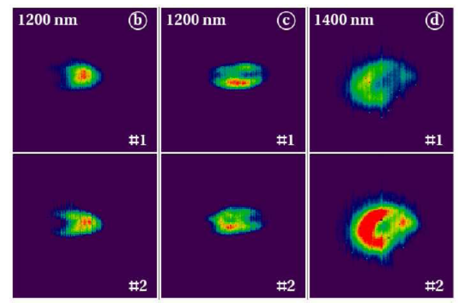
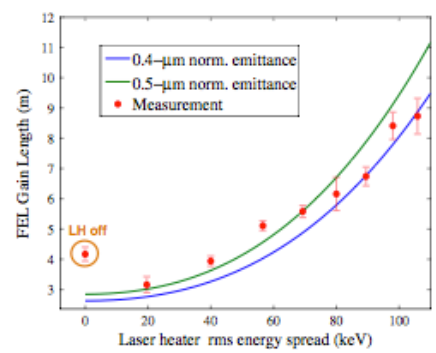


Figure 7: Selection of transverse profiles at different on-crest configurations and various spectral filters. The dimensions of the images are $2\times2\,\mathrm{mm}^2$

Perturbed phase-space distribution suppresses gain in x-ray free-electron lasers



PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **13**, 020703 (2010)

Measurements of the linac coherent light source laser heater and its impact on the x-ray free-electron laser performance

Motivation for this Work: The Longitudinal Space-Charge Amplifier

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 13, 110701 (2010)

Using the longitudinal space charge instability for generation of vacuum ultraviolet and x-ray radiation

E. A. Schneidmiller and M. V. Yurkov

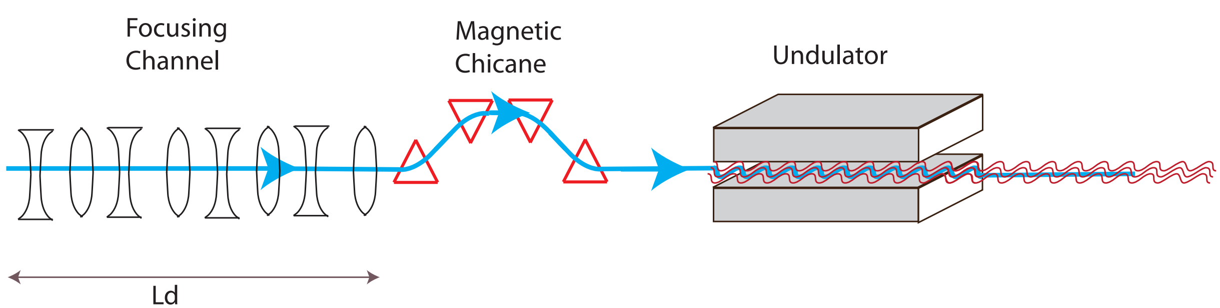
Deutsches Elektronen-Synchrotron (DESY), Notkestrasse 85, D-22607 Hamburg, Germany

(Received 1 April 2010; published 13 November 2010)

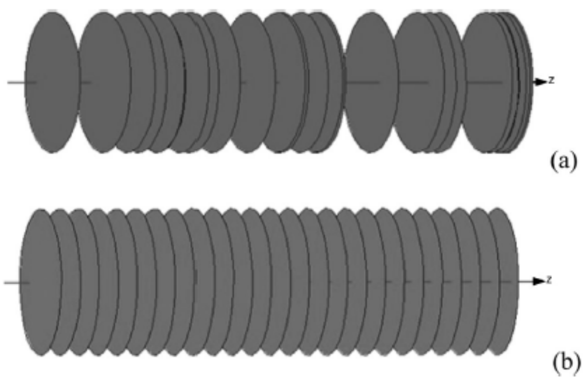
-Inexpensive (no undulator involved in the gain process)

-Lower beam-quality requirements (advanced electron sources? PWFA or DWFA?)

-Alternative to FEL when broad-band is required (e.g. attosecond pulses)



MOTIVATION FOR THIS WORK



PRL 102, 154801 (2009) PHYSICAL REVIEW LETTERS week ending 17 APRIL 2009

Collective-Interaction Control and Reduction of Optical Frequency Shot Noise in Charged-Particle Beams

A. Gover and E. Dyunin
Tel-Aviv University, Faculty of Engineering, Department of Physical-Electronics, Tel-Aviv, Israel
(Received 15 December 2008; published 14 April 2009)

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 14, 060710 (2011)

Analysis of shot noise suppression for electron beams

Daniel Ratner
Department of Applied Physics, Stanford University, Stanford, California 94305, USA

Zhirong Huang and Gennady Stupakov
SLAC, Menlo Park, California 94309, USA
(Received 14 August 2010; published 24 June 2011)

-For a cold laminar beam space-charge suppresses shot noise after a ¼ plasma oscillation

-Energy-spread limits amount of suppression (Ratner, PRSTAB 2011 and Kim FEL 2011)

-What happens with finite emittance?

Approach to the Problem

We are interested in analyzing the evolution of microbunching under the effect of longitudinal space-charge including:

-Emittance
-Betatron Motion
-Energy Spread
-Edge Effects
(Previous work on the subject only included edge-effects).

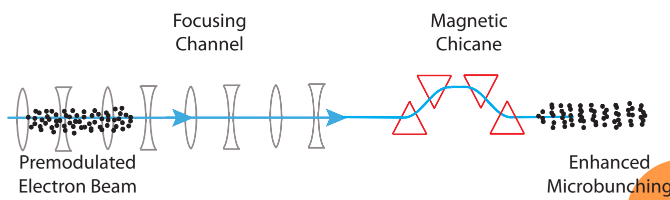
1) Eigenvalue/Eigenmode Analysis:

$$\mathscr{E}_z = E_z(\vec{x})e^{ik_z z - i\frac{\omega\tau}{c}}$$

2) Initial Value Problem

$$f_1(\tau) = \sum_n f_n e^{-i\frac{\omega_n \tau}{c}} \frac{\langle f_n^\dagger, f_{1,0}\rangle}{\langle f_n^\dagger, f_n\rangle}.$$

3) Application to specific microbunching problems



6-Dimensional Dispersion Relation for Space-Charge Modes

$$\begin{cases} \partial_\tau f_1 + \vec{\beta}_\perp \cdot \vec{\nabla}_{\vec{x}} f_1 - k_\beta^2 \vec{x} \cdot \vec{\nabla}_{\vec{\beta}_\perp} f_1 + \\ i k_z \dot{z} f_1 + \frac{e\mathcal{E}_z}{\gamma_0 m c^2} \partial_\eta f_0 = 0 \\ (\nabla_\perp^2 - \frac{k_z^2}{\gamma^2}) \frac{\mathcal{E}_z}{-\frac{ik_z}{\gamma}} = -\frac{e}{\gamma_0 \epsilon_0} \int f_1 d\eta d^2\vec{\beta}_\perp \end{cases}$$

$$\mathcal{E}_z = E_z(\vec{x}) e^{i(k_z z - \omega t)}$$

$$f = f_0 + f_1 e^{i k_z z}$$

6-Dimensional Dispersion Relation for Space-Charge Modes

$$\partial_\tau f_1 + \vec{\beta}_\perp \cdot \vec{\nabla}_{\vec{x}} f_1 - k_\beta^2 \vec{x} \cdot \vec{\nabla}_{\vec{\beta}_\perp} f_1 + ik_z \dot{z} f_1 + \frac{e\mathcal{E}_z}{\gamma_0 mc^2} \partial_\eta f_0 = 0$$

$$(\nabla_\perp^2 - \frac{k_z^2}{\gamma^2}) \frac{\mathcal{E}_z}{-\frac{ik_z}{\gamma}} = -\frac{e}{\gamma_0 \epsilon_0} \int f_1 d\eta d^2\vec{\beta}_\perp$$

Coupled Vlasov-Poisson Equations

$$\mathcal{E}_z = E_z(\vec{x}) e^{i(k_z z - \omega t)}$$

Propagating Space-Charge Mode

$$f = f_0 + f_1 e^{ik_z z}$$

6-Dimensional Dispersion Relation for Space-Charge Modes

$$\partial_\tau f_1 + \vec{\beta}_\perp \cdot \vec{\nabla}_{\vec{x}} f_1 - k_\beta^2 \vec{x} \cdot \vec{\nabla}_{\vec{\beta}_\perp} f_1 + ik_z \dot{z} f_1 + \frac{e\mathcal{E}_z}{\gamma_0 mc^2}\partial_\eta f_0 = 0$$

$$(\nabla_\perp^2 - \frac{k_z^2}{\gamma^2})\frac{\mathcal{E}_z}{-\frac{ik_z}{\gamma}} = -\frac{e}{\gamma_0\epsilon_0}\int f_1 d\eta d^2\vec{\beta}_\perp$$

Coupled Vlasov-Poisson Equations

$$\mathcal{E}_z = E_z(\vec{x})e^{i(k_z z - \omega t)}$$

$$f = f_0 + f_1 e^{ik_z z}$$

Matched Gaussian Distribution

$$f_0 = n_0 e^{-\frac{\vec{x}^2}{2\sigma_x^2} - \frac{\vec{\beta}_\perp^2}{2\sigma_x^2 k_\beta^2} - \frac{\eta^2}{2\sigma_\eta^2}} / (2\pi)^{3/2}\sigma_x^2 k_\beta^2 \sigma_\eta$$

6-Dimensional Dispersion Relation for Space-Charge Modes

$$\partial_\tau f_1 + \vec{\beta}_\perp \cdot \vec{\nabla}_{\vec{x}} f_1 - k_\beta^2 \vec{x} \cdot \vec{\nabla}_{\vec{\beta}_\perp} f_1 + ik_z \dot{z} f_1 + \frac{e\mathcal{E}_z}{\gamma_0 mc^2} \partial_\eta f_0 = 0$$

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Coupled Vlasov-Poisson Equations

$$\mathcal{E}_z = E_z(\vec{x}) e^{i(k_z z - \omega t)}$$

$$f = f_0 + f_1 e^{ik_z z}$$

Phase-Space Perturbation

6-Dimensional Dispersion Relation for Space-Charge Modes

Implicit Solution to Vlasov Equation

$$f_1 = -e^{ik_z z - \frac{i\omega\tau}{c}} \frac{e}{\gamma mc^2} \partial_\eta f_0 \int_{-\infty}^{0} e^{-i\frac{\omega_n \tau}{c} + ik_z \dot{z}\tau} E_z\left(\vec{x}_+(\tau)\right) d\tau$$

Substitute into Poisson's

$$\left(\frac{1}{D^2}\nabla_\perp^2 - 1\right) E_z = -\int E_z(\vec{X}')\Pi(\vec{X},\vec{X}')d^2\vec{X}'$$

$$\Pi(\vec{X},\vec{X}') = \int_{-\infty}^{0} \frac{Te^{-\frac{(K_\gamma T)^2}{2} - i\Omega T} e^{-\left(\vec{X}^2 + \vec{X}'^2 - 2\vec{X}\cdot\vec{X}' \cos K_\beta T\right)\frac{(1+iK_\epsilon T)}{2\sin^2 K_\beta T}}}{2\pi \sin^2 K_\beta T} dT.$$

Marinelli, Hemsing, Rosenzweig
Phys. Plasmas 18, 103105 (2011); doi:10.1063/1.3638139

6-Dimensional Dispersion Relation for Space-Charge Modes

Implicit Solution to Vlasov Equation

$$f_1 = -e^{ik_z z - \frac{i\omega\tau}{c}} \frac{e}{\gamma m c^2} \partial_\eta f_0 \int_{-\infty}^{0} e^{-i\frac{\omega_n \tau}{c} + ik_z \dot{z}\tau} E_z\left(\vec{x}_+(\tau)\right) d\tau$$

Substitute into Poisson's

Dispersion relation has to be solved in E_z and Ω with:

$$\Omega = \omega/\omega_p$$

$$\omega_p^2 = \frac{n_0 e^2}{\gamma^3 m \epsilon_0}$$

$$\left(\frac{1}{D^2}\nabla_\perp^2 - 1\right) E_z = -\int E_z(\vec{X}')\Pi(\vec{X},\vec{X}')d^2\vec{X}'$$

$$\Pi(\vec{X},\vec{X}') = \int_{-\infty}^{0} \frac{T e^{-\frac{(K_\gamma T)^2}{2} - i\Omega T} e^{-\left(\vec{X}^2 + \vec{X}'^2 - 2\vec{X}\cdot\vec{X}' \cos K_\beta T\right)\frac{(1+iK_\epsilon T)}{2\sin^2 K_\beta T}}}{2\pi \sin^2 K_\beta T} dT.$$

Marinelli, Hemsing, Rosenzweig
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6-DIMENSIONAL DISPERSION RELATION FOR SPACE-CHARGE MODES

Implicit Solution to Vlasov Equation

$$f_1 = -e^{ik_z z - \frac{i\omega\tau}{c}} \frac{e}{\gamma m c^2} \partial_\eta f_0 \int_{-\infty}^{0} e^{-i\frac{\omega_n \tau}{c} + ik_z \dot{z}\tau} E_z\left(\vec{x}_+(\tau)\right) d\tau$$

Substitute into Poisson's

Dispersion relation expressed in terms of 4 scaling parameters

$$\left(\frac{1}{D^2}\nabla_\perp^2 - 1\right) E_z = -\int E_z(\vec{X}')\Pi(\vec{X},\vec{X}')d^2\vec{X}'$$

$$\Pi(\vec{X},\vec{X}') = \int_{-\infty}^{0} \frac{T e^{-\frac{(K_\gamma T)^2}{2} - i\Omega T} e^{-\left(\vec{X}^2 + \vec{X}'^2 - 2\vec{X}\cdot\vec{X}'\cos K_\beta T\right)\frac{(1+iK_\epsilon T)}{2\sin^2 K_\beta T}}}{2\pi \sin^2 K_\beta T} dT.$$

Marinelli, Hemsing, Rosenzweig
Phys. Plasmas 18, 103105 (2011); doi:10.1063/1.3638139

Dimensionless Scaling Parameters

Energy spread parameter

$$K_\gamma = k_z c \sigma_\eta / \omega_p \gamma^2 = \frac{\sigma_{v_z} \tau_p}{\lambda}$$

Longitudinal displacement due to energy spread in a 1-d plasma period / microbunching wavelength.

Emittance parameter

$$K_\epsilon = k_z c (k_\beta \sigma_x)^2 / 2\omega_p = \frac{\sigma^\epsilon_{v_z} \tau_p}{\lambda}$$

Longitudinal displacement due to transverse emittance in a 1-D plasma period / microbunching wavelength.

Cold beam limit: K_γ, K_ε << 1

3-D parameter

$$D = k_z \sigma_x / \gamma$$

Transverse beam size/ microbunching wavelength in the rest frame.

Focusing parameter

$$K_\beta = k_\beta c / \omega_p$$

Betatron frequency/1-D plasma frequency

edge effects are negligible if $D \gg 1$

Transverse motion negligible if: $K_\beta \ll 1$
(laminar beam limit)

One Dimensional Limit

The one dimensional limit is approached by taking:

D>>1 , K_β << 1, K_ε << 1

Modes are fully degenerate (all eigenmodes have the same eigenvalue). Dispersion relation reduces to the well known 1-D plasma oscillation dispersion relation for a warm plasma (Landau/Jackson):

$$1 - \frac{1}{2K_\gamma^2} Z'\left(\frac{\Omega}{\sqrt{2}K_\gamma}\right) = 0$$

$$Z(\zeta) = 2ie^{-\zeta^2}\int_{-\infty}^{i\zeta} e^{-x^2}dx$$

In the 1-D limit for a cold beam (*D>>1* , K_β << 1, K_ε << 1 and K_γ << 1) we get the well established result:

$$\Omega^2 = 1$$

or:

$$\omega = \pm\omega_p$$

Initial Value Problem

Solve Poisson equation via Green's function

$$\hat{E}_z(f_1) = \int d\eta d^2\vec{\beta}_\perp d^2\vec{x}' f_1(\vec{x}', \vec{\beta}_\perp, \eta, \tau) E_{sp}(\vec{x} - \vec{x}')$$

Reduce the problem to a Schrodinger-like equation

$$\partial_\tau f_1 = L(f_1)$$

$$L(f_1) = -(\vec{\beta}_\perp \cdot \vec{\nabla}_{\vec{x}} f_1 - k_\beta^2 \vec{x} \cdot \vec{\nabla}_{\vec{\beta}_\perp} f_1 + i k_z \dot{z} f_1 + \frac{e\hat{E}_z(f_1)}{\gamma_0 mc^2} \partial_\eta f_0).$$

Phase-space eigenmodes associated to field eigenmodes

$$\Omega_n, E_n(\vec{X}) \quad \longleftrightarrow \quad \hat{f}_n = -\frac{e}{\gamma mc^2} \partial_\eta f_0 \int_{-\infty}^{0} e^{-i\frac{\omega_n \tau}{c} + i k_z \dot{z}\tau} E_n\left(\vec{x}_+(\tau)\right) d\tau$$

$$L(\hat{f}_n) = -i\frac{\omega_n}{c}\hat{f}_n$$

Initial Value Problem

L is not self-adjoint…
Mode decomposition requires a bi-orthogonal mode expansion

$$f_1(\tau) = \sum_n \hat{f}_n e^{-i\frac{\omega_n \tau}{c}} \frac{\langle \hat{f}_n^\dagger, f_{1,0} \rangle}{\langle \hat{f}_n^\dagger, \hat{f}_n \rangle}.$$

INITIAL VALUE PROBLEM

L is not self-adjoint…
Mode decomposition requires a bi-orthogonal mode expansion

Initial Phase-Space Perturbation

$$f_1(\tau) = \sum_n \hat{f}_n e^{-i\frac{\omega_n \tau}{c}} \frac{\langle \hat{f}_n^\dagger, f_{1,0} \rangle}{\langle \hat{f}_n^\dagger, \hat{f}_n \rangle}.$$

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Adjoint phase-space eigenmodes

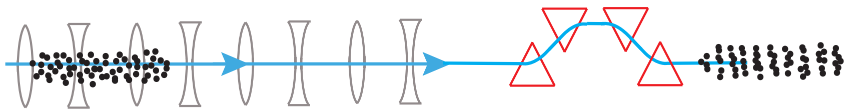
$$\hat{f}_n^\dagger = -\frac{e}{\gamma_0 m c^2} \int_{-\infty}^{0} e^{-i\frac{\omega_n \tau}{c} + i k_z \dot{z} \tau} E_n\left(\vec{x}_-(\tau)\right) d\tau,$$

Microbunching Evolution and Amplification

With our solution to the initial value problem we can discuss specific microbunching problems

$$f_1(\tau) = \sum_n f_n e^{-i\frac{\omega_n \tau}{c}} \frac{\langle f_n^\dagger, f_{1,0}\rangle}{\langle f_n^\dagger, f_n\rangle}.$$

This formula gives the evolution of the initial perturbation under the effect of LSC.
Dispersion in magnetic chicane results in a factor Exp(-ikηR_{56})



Cold beam evolution of microbunching in the drift:

$$\tilde{n} = \tilde{n}_0 \Gamma \cos \omega_p \Omega c t$$

Cold beam gain:

$$\tilde{n} = -\tilde{n}_0 \gamma^2 R_{56} e^{-\frac{(k_z \sigma_\eta R_{56})^2}{2}} \Gamma \frac{\omega_p}{c} \Omega_+ \sin \omega_p \Omega_+ \frac{L_d}{c}$$

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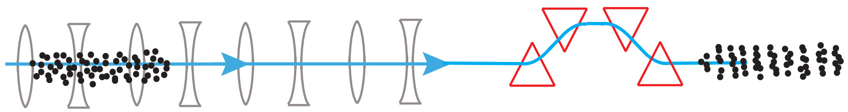

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Plasma oscillation in the drift

Cold beam gain:

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Microbunching Evolution and Amplification

With our solution to the initial value problem we can discuss specific microbunching problems

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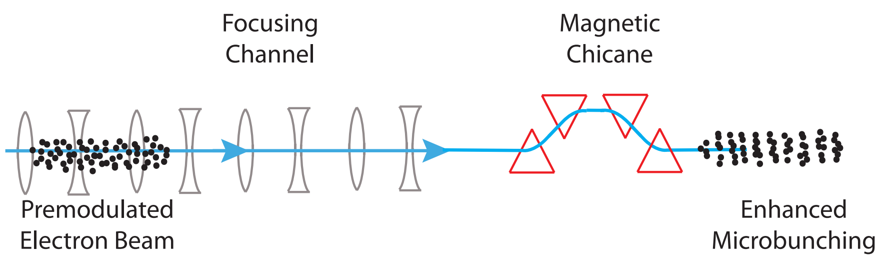


Cold beam evolution of microbunching in the drift:

$$\tilde{n} = \tilde{n}_0 \Gamma \cos \omega_p \Omega ct$$

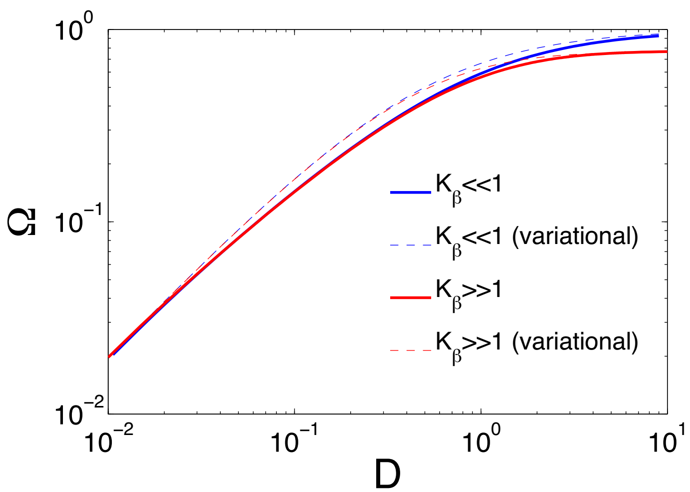
Time derivative of the density modulation at the drift exit!

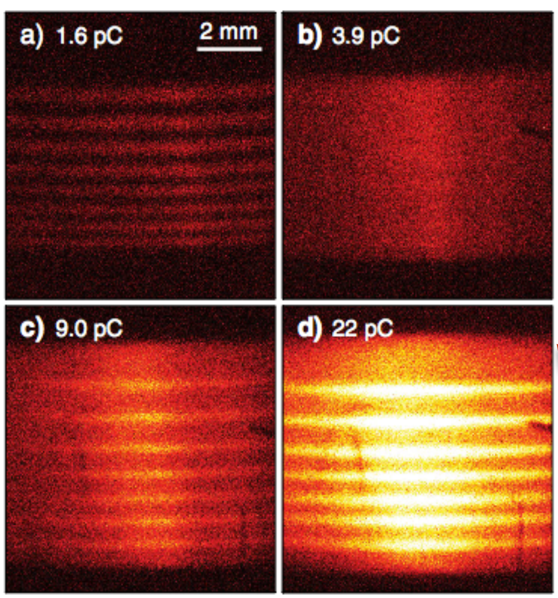
Gain Maximized at ¼ plasma oscillation!

Cold beam gain:

$$\tilde{n} = -\tilde{n}_0 \gamma^2 R_{56} e^{-\frac{(k_z \sigma_\eta R_{56})^2}{2}} \Gamma \frac{\omega_p}{c} \Omega_+ \sin \omega_p \Omega_+ \frac{L_d}{c}$$

Geometrical Effects and Betatron Motion: Plasma Reduction Factor for a Cold Beam





Assume Cold beam limit: K_γ, K_ε << 1.

In the “infinite beam limit” (or short wavelength limit)
D>>1

Ω=1 for a laminar beam (K_β<<1)
Ω≈0.756 for high betatron frequency(K_β>>1)

In the “small beam limit” (or long wavelength limit)
D<<1

Ω≈2D regardless of betatron motion

Reduction due to geometry of the microbunched beam in the rest frame. For for small D you have: $\gamma\lambda >> \sigma_x$

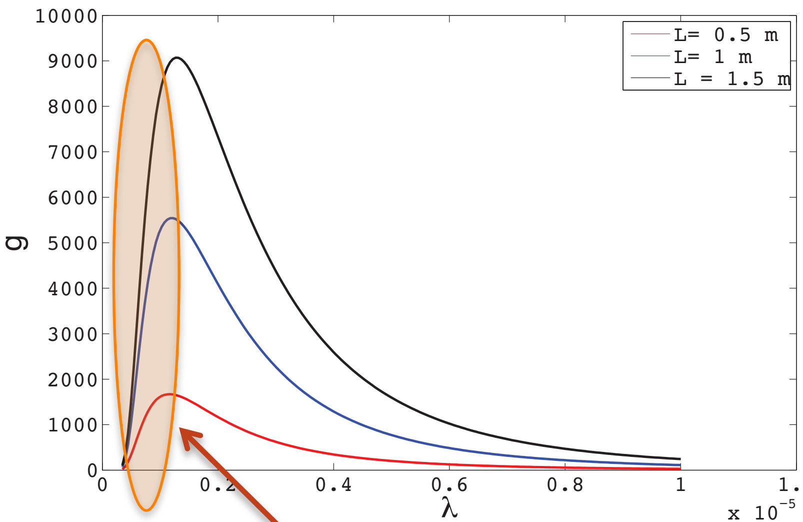
Handy variational formula for laminar beam limit:

$$\Omega = 2D/(1+2D)$$

Experimental validation of the cold-beam results:
Musumeci, Li, Marinelli

Phys. Rev. Lett. 106, 184801 (2011) [4 pages]

Cold Beam Amplification Spectrum



Example: compressed NLCTA beam

- I_{peak} = 500 A
- $\varepsilon = 3 \times 10^{-6}$ mm mrad (slice emittance)
- γ = 240 (E = 120 MeV)
- λ = 800 nm
- $E_{spread} = 5 \times 10^{-5}$
- L_d = 0.5 m/1 m /1.5 m

$$\tilde{n} = -\tilde{n}_0 \gamma^2 R_{56} e^{-\frac{(k_z \sigma_\eta R_{56})^2}{2}} \Gamma \frac{\omega_p}{c} \Omega_+ \sin \omega_p \Omega_+ \frac{L_d}{c}$$

Short wavelength behavior dominated by energy-spread

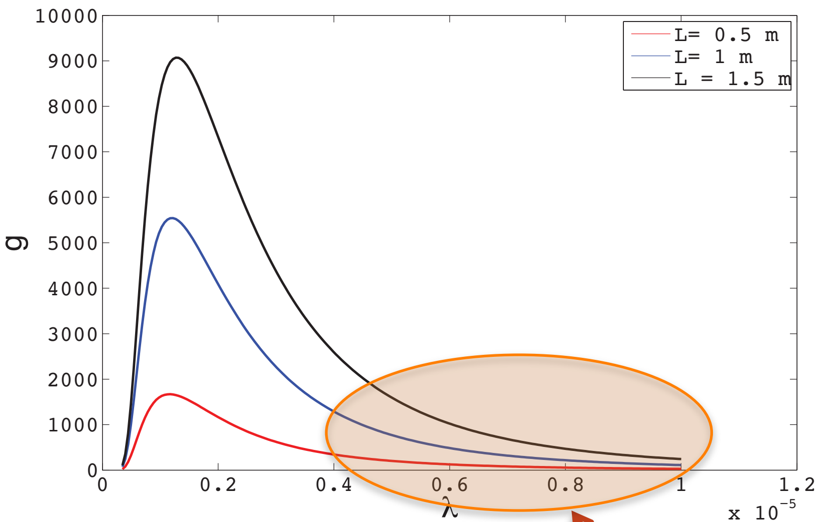
$$g \sim e^{-(k\sigma_\eta R_{56})^2}$$

$$\Omega \approx \frac{2D}{e^{-\alpha D} + \beta D}$$

$$\beta = 2/0.763$$

$$\alpha = 2 - \beta$$

Cold Beam Amplification Spectrum



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Long wavelength behavior dominated by edge effects

$$\Omega \sim 1/\lambda$$

$$g \sim 1/\lambda^2$$ at ~ ¼ plasma period

$$g \sim 1/\lambda^4$$ at << ¼ plasma period

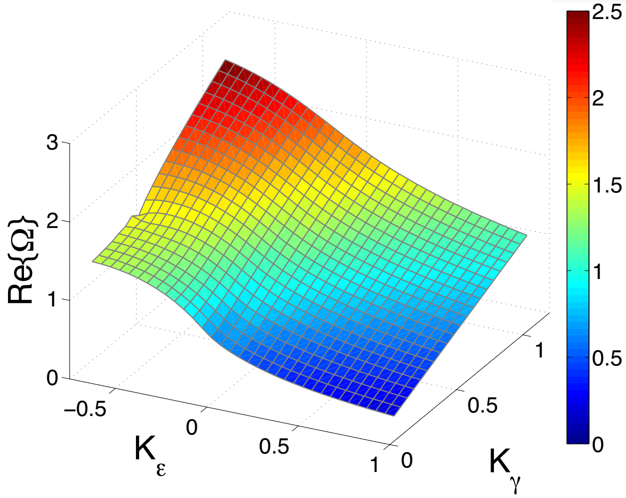
$$\Omega \approx \frac{2D}{e^{-\alpha D} + \beta D}$$

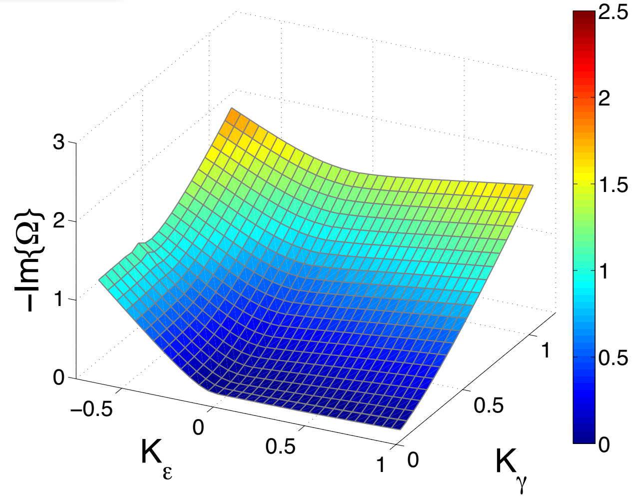
$$\beta = 2/0.763$$

$$\alpha = 2 - \beta$$

Emittance and Energy Spread Effects

Example:
$D=1\ K_\beta >>1$



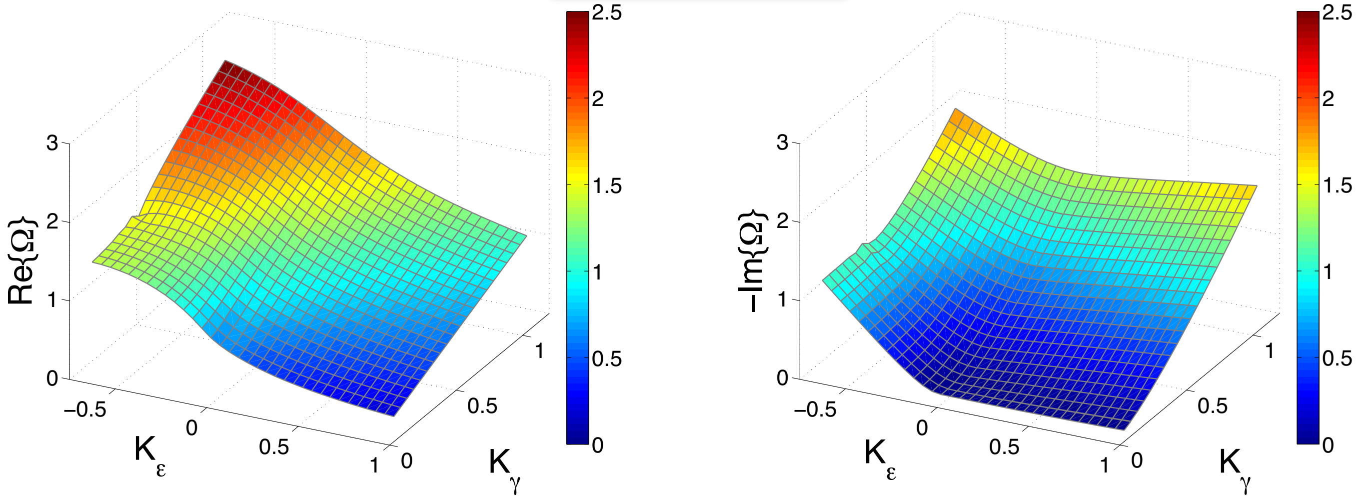


Longitudinal thermal motion induced by energy spread and transverse emittance gives rise to an exponential damping process (Landau damping).

$$\dot{z} = \eta/\gamma^2 - (k_\beta^2 \vec{x}^2 + \vec{\beta}_\perp^2)/4$$

Emittance and Energy Spread Effects

Example:
D=1 K_β >>1



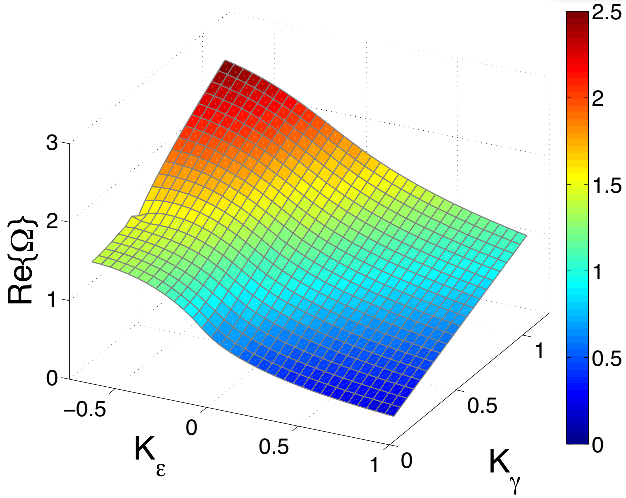
Longitudinal thermal motion induced by energy spread and transverse emittance gives rise to an exponential damping process (Landau damping).

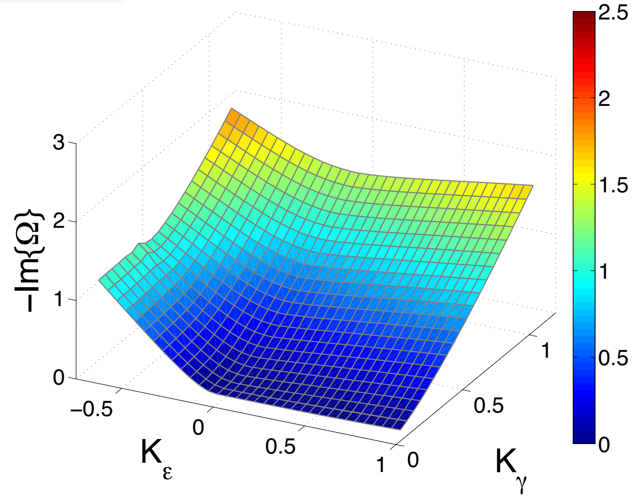
$$\dot{z} = \eta/\gamma^2 - (k_\beta^2 \vec{x}^2 + \vec{\beta}_\perp^2)/4$$

Symmetric distribution:
Isotropic Landau damping

Emittance and Energy Spread Effects

Example:
$D=1\ K_\beta >>1$





Longitudinal thermal motion induced by energy spread and transverse emittance gives rise to an exponential damping process (Landau damping).

$$\dot{z} = \eta/\gamma^2 - (k_\beta^2 \vec{x}^2 + \vec{\beta}_\perp^2)/4$$

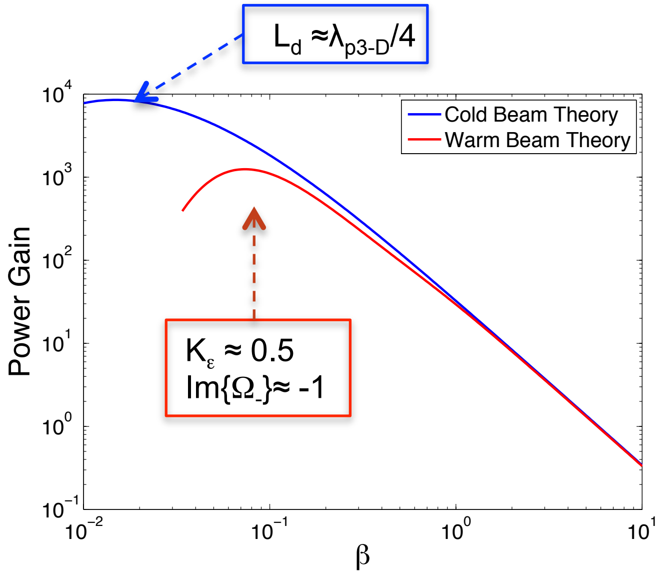
Always negative!
Asymmetric distribution:
Anisotropic Landau damping

Emittance induced Landau damping sets the optimum beam size for space-charge experiments since:

$\omega_p \sim 1/\sigma_x$ and $K_\varepsilon \sim 1/\sigma_x$

Increasing the density by focusing comes at the expense of increasing longitudinal velocity spread!

3-D and Thermal Effects in the Longitudinal Space-Charge Amplifier



Example: compressed NLCTA beam

- I_{peak} = 500 A
- $\varepsilon = 3 \times 10^{-6}$ mm mrad (slice emittance)
- γ = 240 (E = 120 MeV)
- λ = 800 nm
- $E_{spread} = 5 \times 10^{-5}$
- L_d = 1 m

1-D Plasma frequency scales like:

$$\omega_p \sim 1/\sigma_x \sim 1/\beta^{1/2}$$

Emittance parameter scales like:

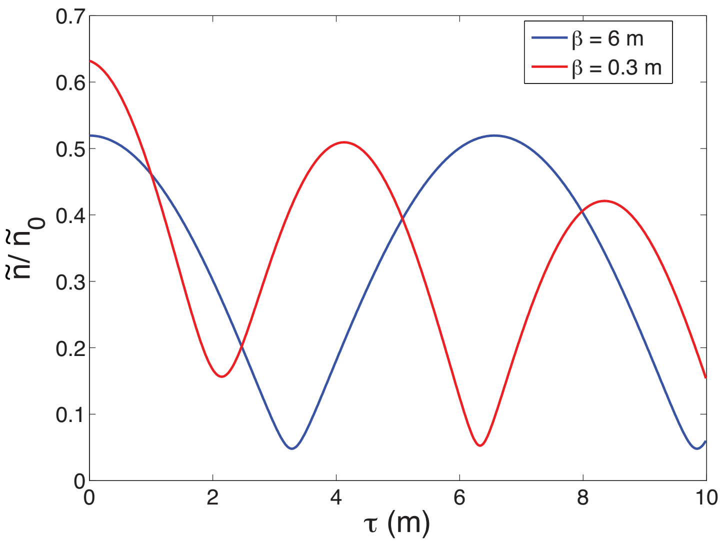
$$K_\varepsilon \sim 1/\sigma_x \sim 1/\beta^{1/2}$$

Stronger focusing enhances the collective response but increases thermal effects due to emittance!
Gain has an optimum when the two effects balance each other!

For the cold beam case, gain increases with stronger focusing until the drift length is equal to ¼ plasma period.

$$\tilde{n} = -\tilde{n}_0 \gamma^2 R_{56} e^{-\frac{(k_z \sigma_\eta R_{56})^2}{2}} \Gamma \frac{\omega_p}{c} \Omega_+ \sin \omega_p \Omega_+ \frac{L_d}{c}$$

Emittance Effects in Noise Suppression

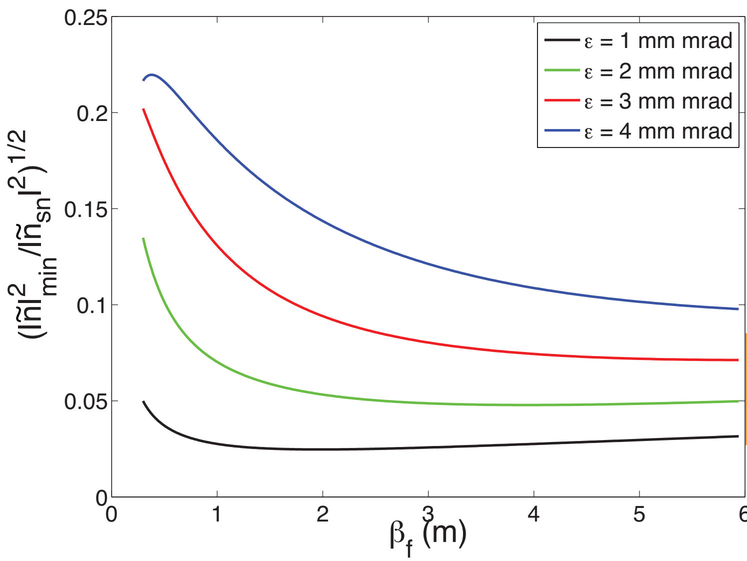


Emittance induces two effects:

-damping of backward component
- >0 minimum microbunching (coupling of the two waves is different…)

Minimum vale of microbunching grows with growing emittance and decreasing beta-function

Fundamental limitation of noise suppression schemes…



Higher Order Modes and Plasma-Betatron Beatwaves

Azimuthal mode expansion of dispersion relation

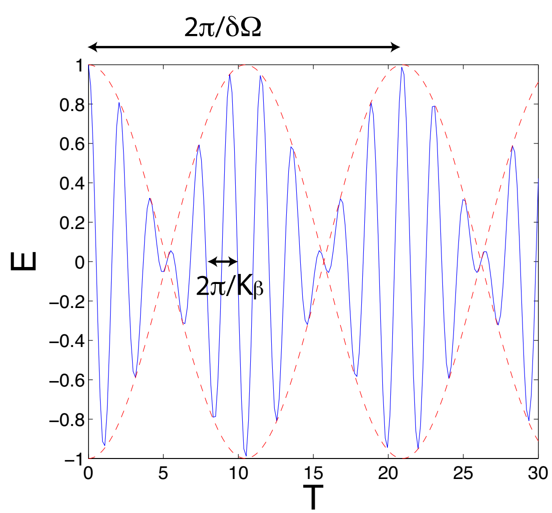
$$E_z = E_m(R)e^{im\theta}$$

+ cold beam limit:

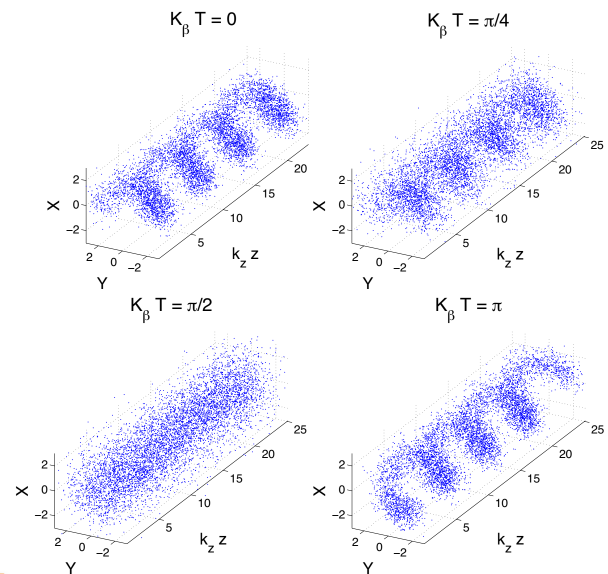
$$\hat{E}_m(Q) = \int_0^\infty T_m(Q,Q')\hat{E}_m(Q')Q'dQ'$$

$$T_m(Q,Q') = \frac{e^{-\frac{Q^2+Q'^2}{2}}}{\left(1+\frac{Q^2}{D^2}\right)} \sum_n I_{\frac{m+n}{2}}\left(\frac{QQ'}{2}\right) I_{\frac{m-n}{2}}\left(\frac{QQ'}{2}\right) \frac{1}{(\Omega - nK_\beta)^2}$$

For $K_\beta >> 1$ $\Omega = hK_\beta \pm \delta\Omega$ with $\delta\Omega << K_\beta$
For $h \neq 0$ beat between betatron and plasma oscillations.



Note:
for an emittance dominated beam odd m modes only exist as beatwaves…



Evolution of an even/odd m charge perturbation under transverse focusing composed of even/odd harmonics of betatron oscillation.
(example: m=1)

Higher Order Modes and Plasma-Betatron Beatwaves

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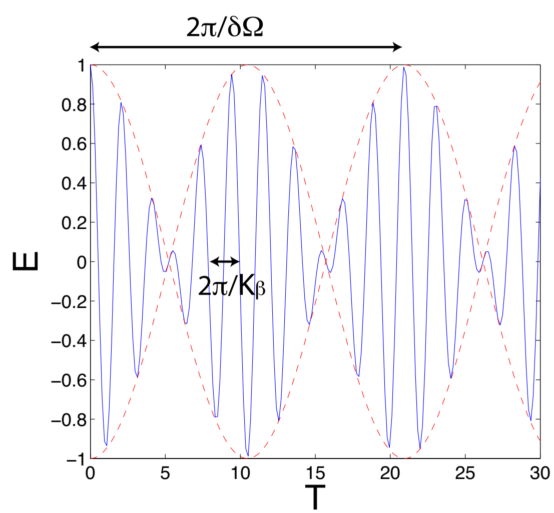
Sum performed over even/odd harmonics for even/odd m
Dispersion relation highly peaked around $\Omega \approx nK_\beta$ (cold beam limit)

For $K_\beta >> 1$ $\Omega = hK_\beta \pm \delta\Omega$ with $\delta\Omega << K_\beta$
For $h \neq 0$ beat between betatron and plasma oscillations.



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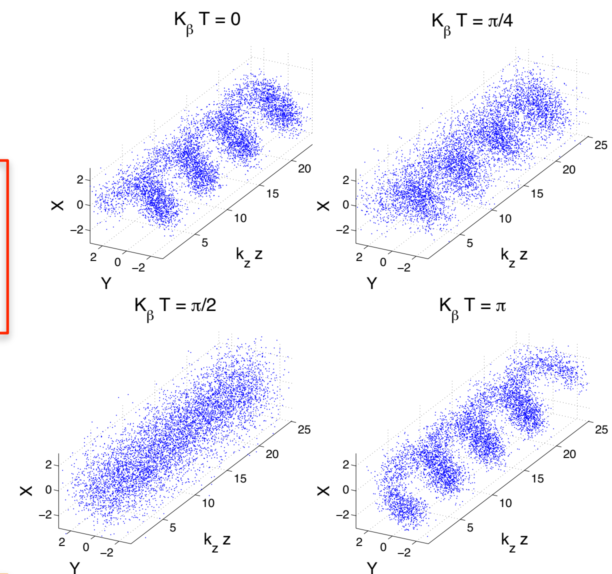
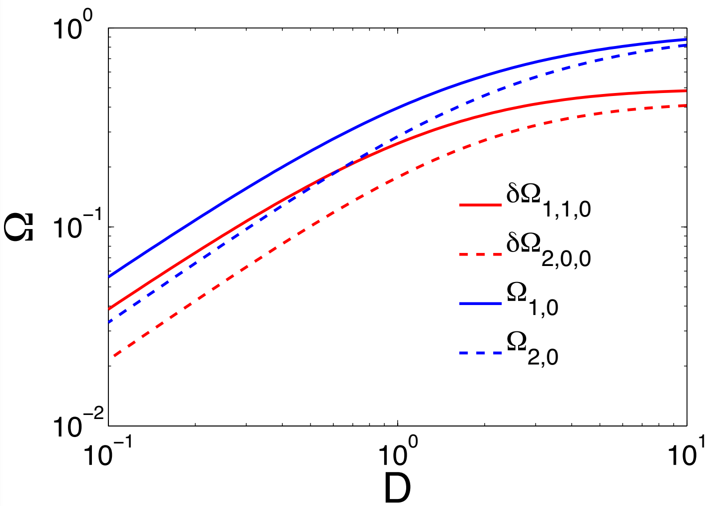


Evolution of an even/odd m charge perturbation under transverse focusing composed of even/odd harmonics of betatron oscillation.
(example: m=1)

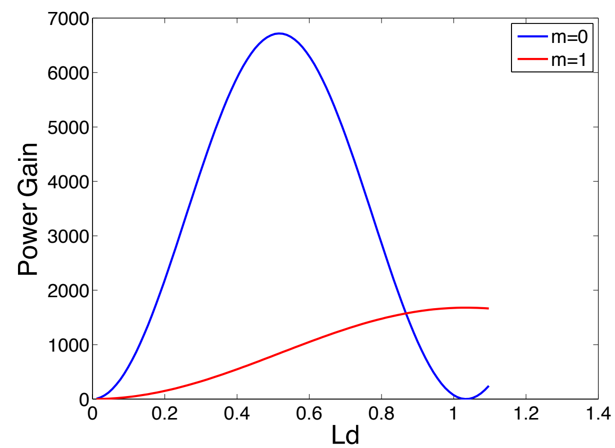
Higher Order Modes in LSCA

suppression of higher order modes due to transverse focusing.



The reduced eigenvalue δΩ describes the collective physics of the system and it's independent of K_β.

Suppress or amplify higher order modes in a LSCA

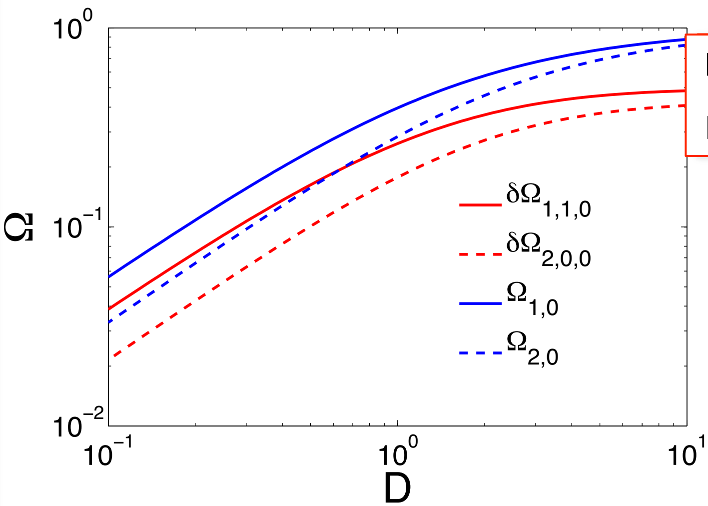


$$G_{m=0} = \left(\frac{\omega_p}{c}\gamma^2 R_{56} e^{-\frac{K_\phi^2}{2}} \Gamma_0 \Omega_0 \sin\left(\Omega_0 \frac{\omega_p}{c} L_d\right)\right)^2$$

$$G_{m=1} = \left(\frac{\omega_p}{c}\gamma^2 R_{56} e^{-\frac{K_\phi^2}{2}} \Gamma_1 \delta\Omega_1 \sin\left(\delta\Omega_1 \frac{\omega_p}{c} L_d\right) \cos\left(K_\beta \frac{\omega_p}{c} L_d\right)\right)^2$$

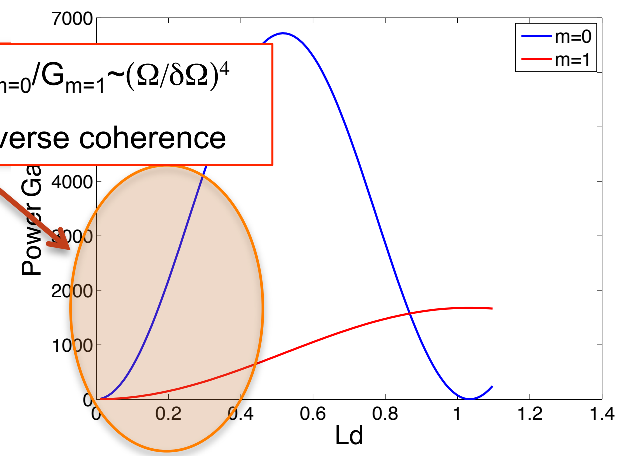
Higher Order Modes in LSCA

suppression of higher order modes due to transverse focusing.



The reduced eigenvalue δΩ describes the collective physics of the system and it's independent of K_{β}.

Suppress or amplify higher order modes in a LSCA



Ld << $\lambda_p/4$ $G_{m=0}/G_{m=1} \sim (\Omega/\delta\Omega)^4$

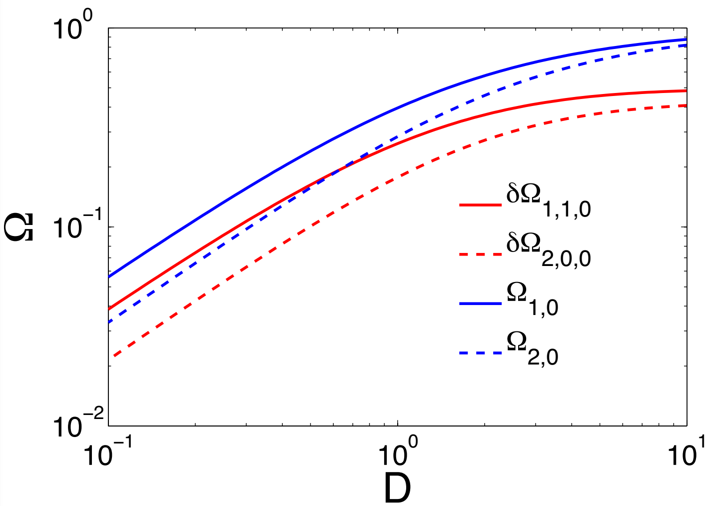
Increased transverse coherence

$$G_{m=0} = \left(\frac{\omega_p}{c} \gamma^2 R_{56} e^{-\frac{K_\phi^2}{2}} \Gamma_0 \Omega_0 \sin\left(\Omega_0 \frac{\omega_p}{c} L_d\right) \right)^2$$

$$G_{m=1} = \left(\frac{\omega_p}{c} \gamma^2 R_{56} e^{-\frac{K_\phi^2}{2}} \Gamma_1 \delta\Omega_1 \sin\left(\delta\Omega_1 \frac{\omega_p}{c} L_d\right) \cos\left(K_\beta \frac{\omega_p}{c} L_d\right) \right)^2$$

Higher Order Modes in LSCA

suppression of higher order modes due to transverse focusing.

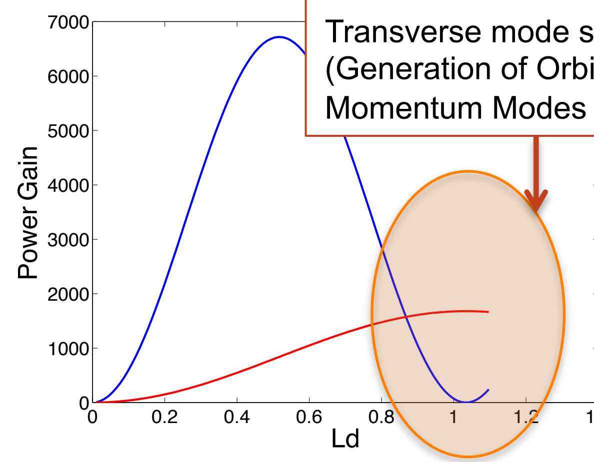


The reduced eigenvalue $\delta\Omega$ describes the collective physics of the system and it's independent of K_β.

Suppress or amplify higher order modes in a LSCA

$L_d \approx \lambda_p/2$ $\quad G_{m=0} \ll G_{m=1}$

Transverse mode selection! (Generation of Orbital Angular Momentum Modes in FELs)



$$G_{m=0} = \left(\frac{\omega_p}{c}\gamma^2 R_{56} e^{-\frac{K_\phi^2}{2}}\Gamma_0\Omega_0 \sin\left(\Omega_0\frac{\omega_p}{c}L_d\right)\right)^2$$

$$G_{m=1} = \left(\frac{\omega_p}{c}\gamma^2 R_{56} e^{-\frac{K_\phi^2}{2}}\Gamma_1\delta\Omega_1 \sin\left(\delta\Omega_1\frac{\omega_p}{c}L_d\right)\cos\left(K_\beta\frac{\omega_p}{c}L_d\right)\right)^2$$

PRL **106,** 164803 (2011) PHYSICAL REVIEW LETTERS week ending 22 APRIL 2011

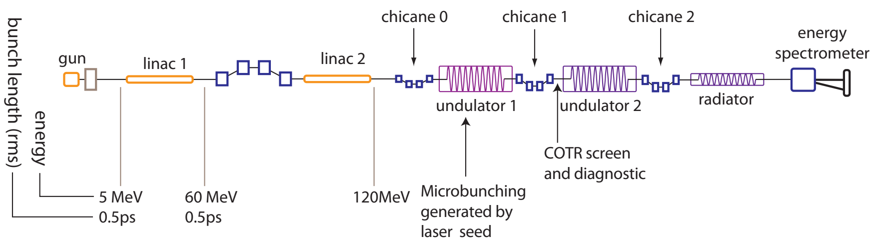
Generating Optical Orbital Angular Momentum in a High-Gain Free-Electron Laser at the First Harmonic

E. Hemsing, A. Marinelli, and J. B. Rosenzweig

Particle Beam Physics Laboratory, Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, California 90095, USA

(Received 6 December 2010; published 22 April 2011)

Work in Progress: LSCA Experiment at NLCTA



-LSCA experiment at NLCTA test facility:
(~summer 2012 ?)

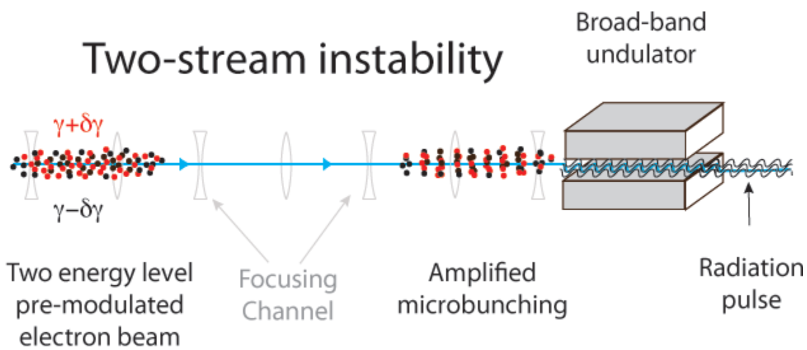
Study amplification and plasma-dynamics of seeded microbunching at 800nm.

NLCTA has a wide tunability of the beam current.
Study dynamics by changing plasma frequency at fixed locations.

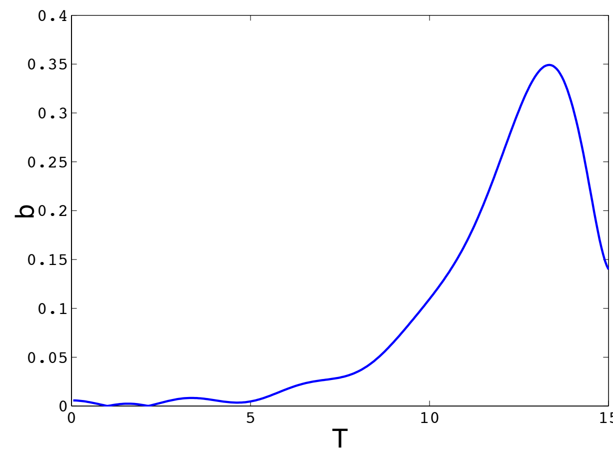
$$\tilde{n} = \tilde{n}_0 \Gamma \cos \omega_p \Omega c t$$

$$\tilde{n} = -\tilde{n}_0 \gamma^2 R_{56} e^{-\frac{(k_z \sigma_\eta R_{56})^2}{2}} \Gamma \frac{\omega_p}{c} \Omega_+ \sin \omega_p \Omega_+ \frac{L_d}{c}$$

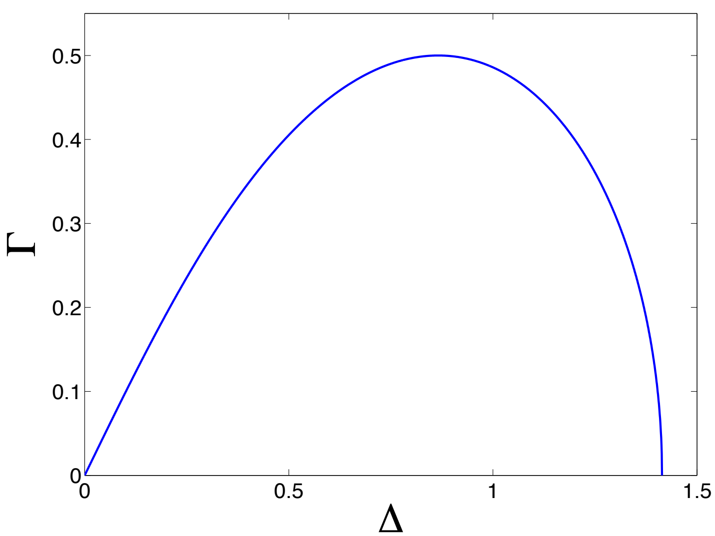
Work in Progress: Two-Stream Amplifier for Ultra-Short Soft X-Ray Pulses



Instability driven by space-charge in a 2-energy level beam



Peak bunching factor vs time



Optimum Gain-length = λ_p/π
Good scaling to soft x-ray wavelengths!

Optimum wavelength:

$$\lambda_{opt} = \frac{2\lambda_p c \Delta\gamma}{\sqrt{3}\gamma^3}$$

Broad RMS bandwidth at saturation ~50%

Work in Progress: Two-Stream Amplifier for Ultra-Short Soft X-Ray Pulses

Amplification of attosecond pulses from HHG in gas!

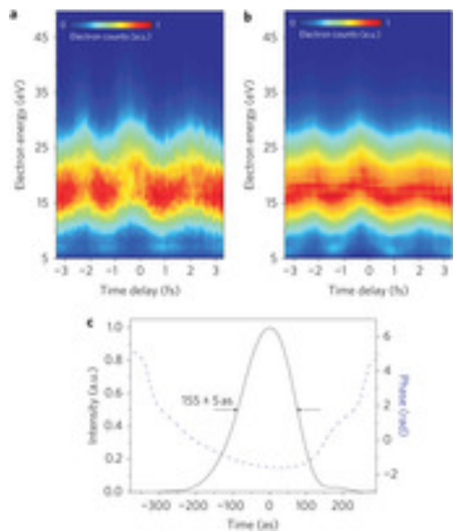
High-energy isolated attosecond pulses generated by above-saturation few-cycle fields

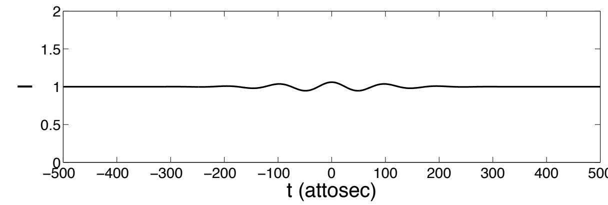
F. Ferrari, F. Calegari, M. Lucchini, C. Vozzi, S. Stagira, G. Sansone & M. Nisoli

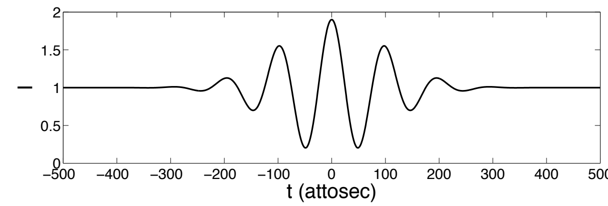
Affiliations | Contributions | Corresponding author

Nature Photonics 4, 875–879 (2010) | doi:10.1038/nphoton.2010.250
Received 08 March 2010 | Accepted 29 September 2010 | Published online 14 November 2010

Theoretical study on-going at 6 nm…
Application to NGLS?



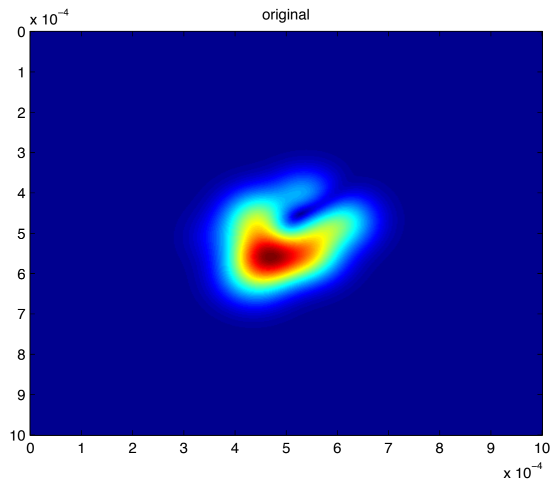


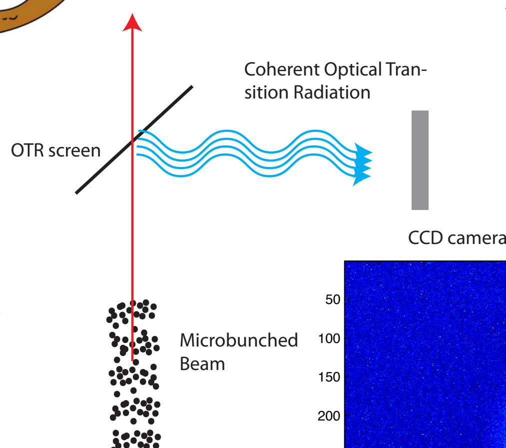


Generation of two-level energy distribution is challenging.

Several methods are being considered…
Two pulse train + off-crest acceleration +chicane
E-SASE scheme + mask?

Reconstruction of Microbunched Beams with Phase Retrieval of COTR





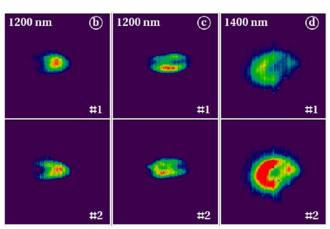
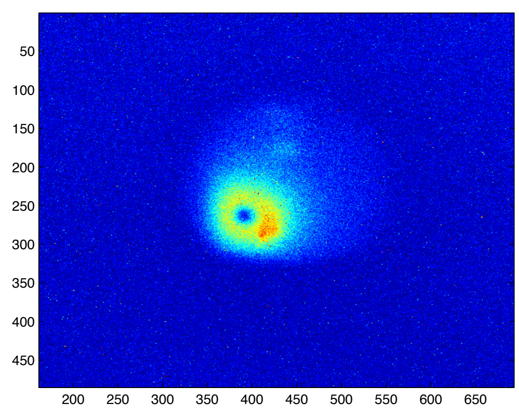


Figure 7: Selection of transverse profiles at different on-crest configurations and various spectral filters. The dimensions of the images are $2 \times 2\,\mathrm{mm}^2$



Reconstruction of Microbunched Beams

Microbunching is often described as a one-dimensional entity:

$$b(k) = \frac{1}{N}\sum e^{-ikz_n} = \frac{1}{N}\int \rho(x,y,z)e^{-ikz}dxdydz$$

By integrating over x-y we lose track of any transverse dependence of the density modulation

In many applications it is necessary to keep record of the transverse distribution:

$$b(x,y,k_z) = \frac{1}{N}\int \rho(x,y,z)e^{-ik_z z}dz$$

Microbunching in X-space

$$B(k_x,k_y,k_z) = \frac{1}{N}\int \rho(x,y,z)e^{-ik_x x-ik_y y-ik_z z}dxdydz$$

Microbunching in K-space

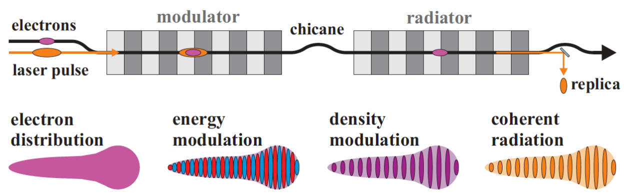
TRANSVERSE OPTICAL REPLICA

Microbunching induced by laser-beam interaction in an undulator.

If $R_{laser} >> R_{beam}$ and $l_{laser} >> l_{beam}$

The microbunching distribution is a replica of the beam's transverse charge distribution.

Diagnostics of compressed beams! (induced microbunching is much larger than MBI, beam image not affected by collective effects…)

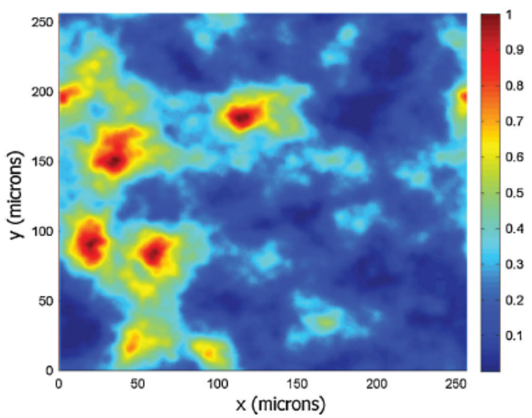


Method for the determination of the three-dimensional structure of ultrashort relativistic electron bunches

Gianluca Geloni, Petr Ilinski, Evgeni Saldin, Evgeni Schneidmiller, Mikhail Yurkov

arXiv:0905.1619v1 [physics.optics]

Three-Dimensional Microbunching: Longitudinal Space-Charge Instability



Transversely incoherent space-charge fields for short wavelengths

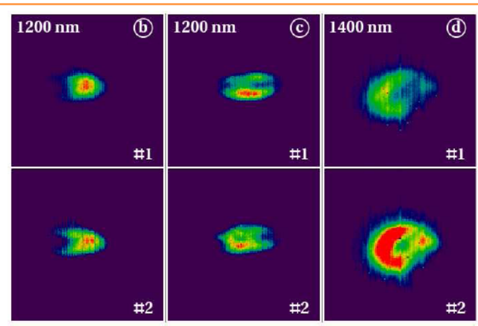
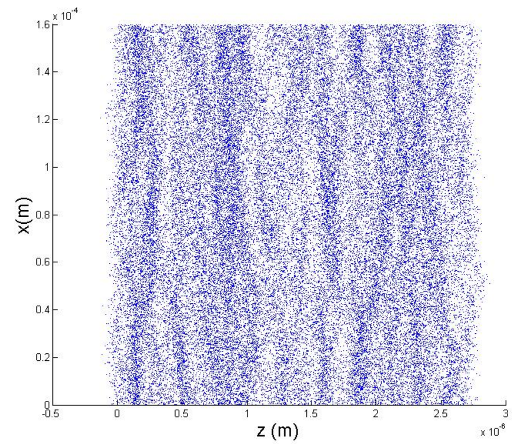


Figure 7: Selection of transverse profiles at different on-crest configurations and various spectral filters. The dimensions of the images are $2\times2\ \mathrm{mm}^2$



Transversely inhomogeneous microbunching

September 2008
SLAC-PUB-13392

THREE-DIMENSIONAL ANALYSIS OF LONGITUDINAL SPACE CHARGE MICROBUNCHING STARTING FROM SHOT NOISE*

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Stanford Linear Accelerator Center, Stanford, CA 94309, USA

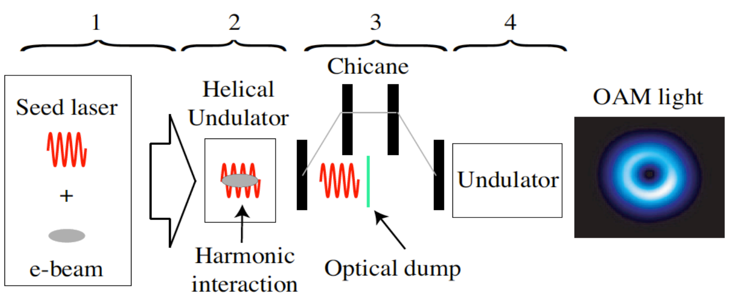
PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 13, 110703 (2010)

Microscopic kinetic analysis of space-charge induced optical microbunching in a relativistic electron beam

Agostino Marinelli[1,2] and James B. Rosenzweig[1]
[1]*Particle Beam Physics Laboratory, Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, California 90095, USA*
[2]*Dipartimento SBAI, Università degli Studi di Roma La Sapienza, Via Antonio Scarpa 14, Rome, 00161, Italy*

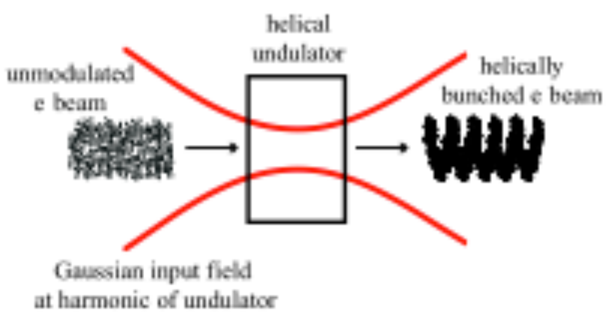
(Received 18 May 2010; published 29 November 2010)

Three-Dimensional Microbunching: Orbital Angular Momentum Modes



Helical charge perturbation from harmonic interaction in a helical undulator:

$$\rho \propto r e^{-\frac{r^2}{2w^2} - i\phi}$$



Phys. Rev. Lett., 106, 164803 (2011)

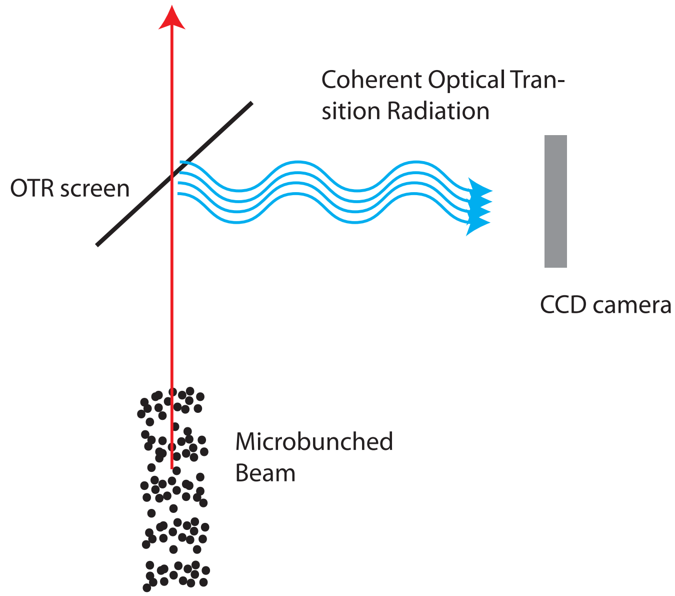
Generation of Optical Orbital Angular Momentum in a High-Gain Free-electron Laser at the First Harmonic

E. Hemsing, A. Marinelli, J. B. Rosenzweig
Particle Beam Physics Laboratory, Department of Physics and Astronomy
University of California Los Angeles, Los Angeles, CA 90095, USA

COTR Diagnostic for Three-Dimensional Microbunching

In these applications, it is interesting to reconstruct the transverse structure of the density modulation in amplitude and phase:

$$b(x,y,k) = \int e^{-ik_z z} \rho(x,y,z) dz$$



Ingredients:

-Narrow bandwidth signal is needed (seeding or bandpass filtering)
-Near or far field imaging?

Near field is hard to interpret:

1) Near field COTR is a convolution between b and the OTR Green's function.

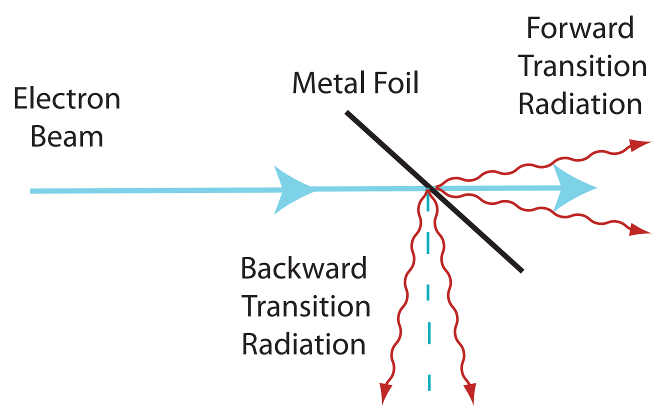
2) Intensity pattern mixes two polarizations!

COTR Diagnostic for Three-Dimensional Microbunching: Far Field Imaging

$$\frac{dP}{d\omega d\Omega}|_{1-part} = \frac{e^2}{4\pi^2 c}\frac{sin^2(\theta)}{(1-\beta^2 cos(\theta))^2}$$

$$\frac{dP}{d\omega d\Omega}|_{coh} = \frac{dP}{d\omega d\Omega}|_{1-part}|B(k_x,k_y,k_z)|^2$$

$$B(k_x,k_y,k_z) = \frac{1}{N}\sum_n e^{-ik_x x_n - ik_y y_n - ik_z z_n}$$



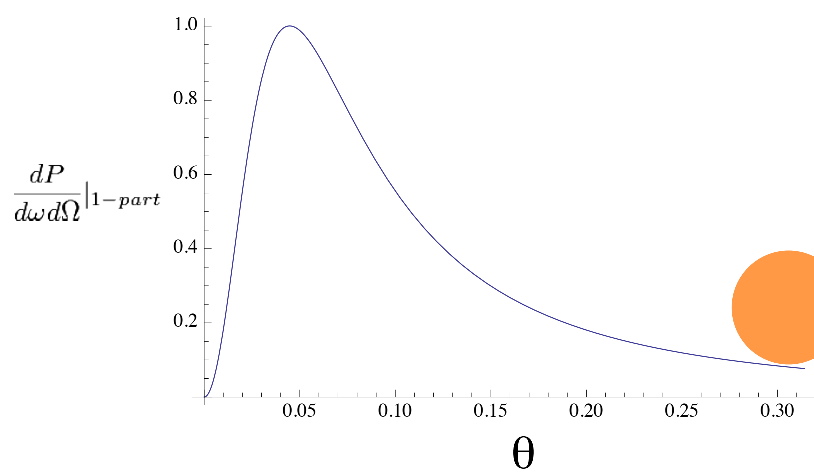
From a single-frequency far-field measurement we can recover

$$|B^2(k_x,k_y,k)|$$

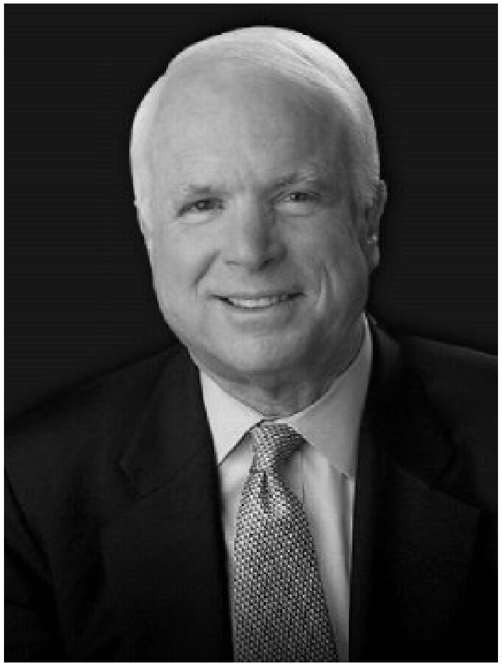
We are interested in

$$b(x,y,k) = \int e^{-ik_x x - ik_y y} B(k_x,k_y,k)\frac{dk_x dk_y}{(2\pi)^2}$$

Phase information on B is needed to recover the signal in x-y space!!



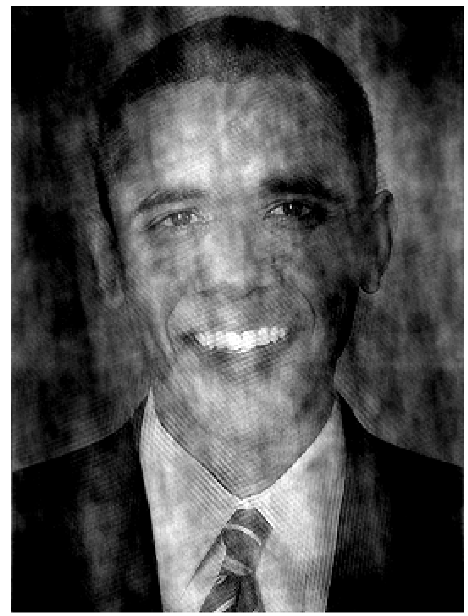
HOW IMPORTANT IS KNOWLEDGE OF PHASE?



$$IFFT\left(\left|Obama_K\right| e^{iArg(Obama_K)}\right) = Obama(X,Y)$$

$$IFFT\left(\left|McCain_K\right| e^{iArg(McCain_K)}\right) = McCain(X,Y)$$

HOW IMPORTANT IS KNOWLEDGE OF PHASE?

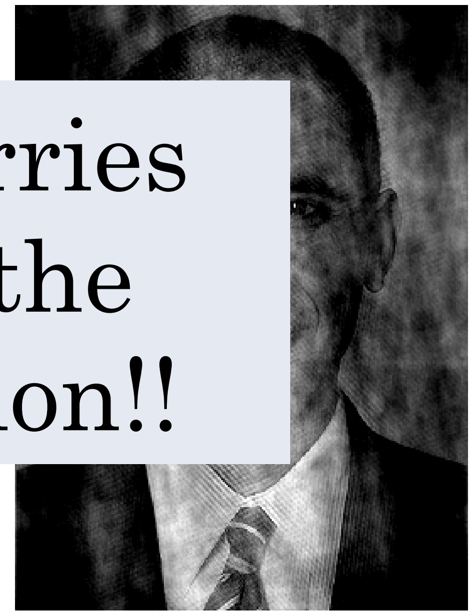


$$IFFT(|Obama_K| e^{iArg(McCain_K)}) \approx McCain(X,Y)$$

$$IFFT(|McCain_K| e^{iArg(Obama_K)}) \approx Obama(X,Y)$$

How Important is Knowledge of Phase?

Phase carries most of the information!!



$$IFFT\left(\left|Obama_K\right| e^{iArg(McCain_K)}\right) \approx McCain(X,Y)$$

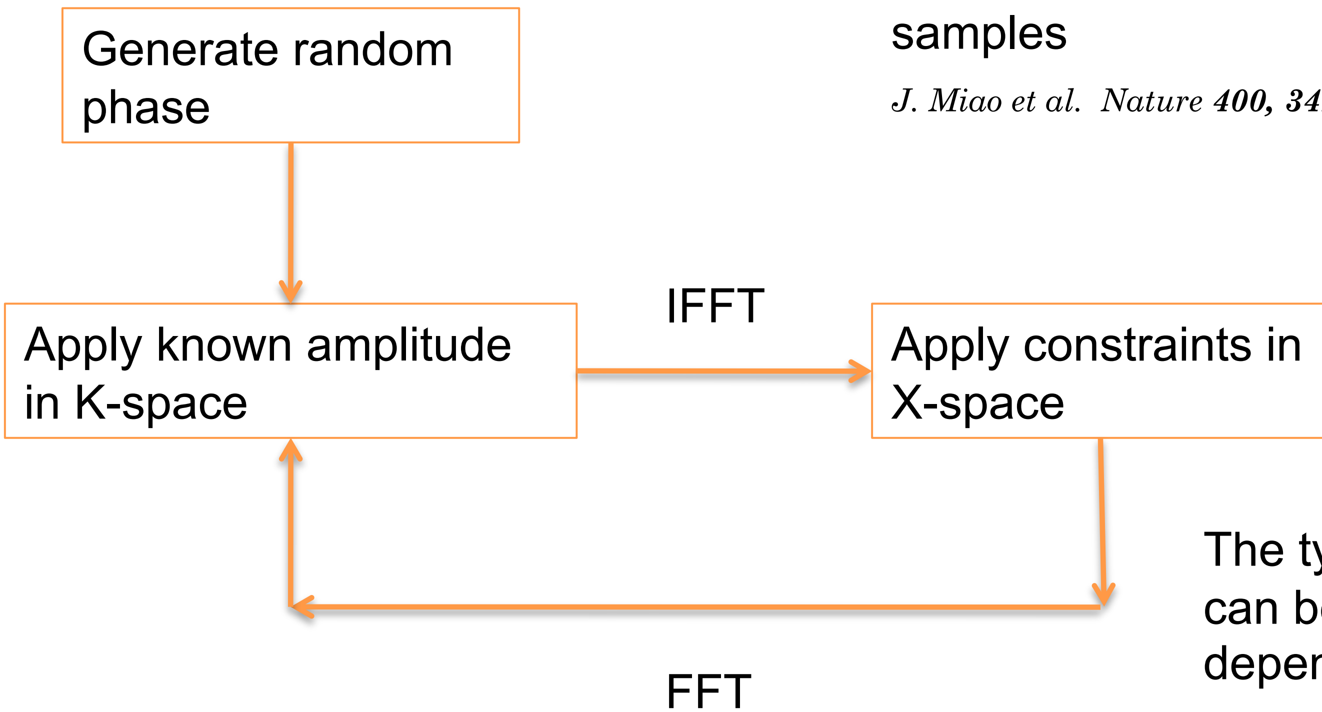
$$IFFT\left(\left|McCain_K\right| e^{iArg(Obama_K)}\right) \approx Obama(X,Y)$$

Phase Retrieval Algorithms

Phase information can be recovered by means of iterative retrieval algorithms.

Customarily used in x-ray microscopy for reconstruction of non-crystalline samples

J. Miao et al. Nature ***400, 342-344 (22 July 1999)***



The type of constraint that can be applied in X-space depends on the experimental implementation of the method

Constraints in X-Y Space

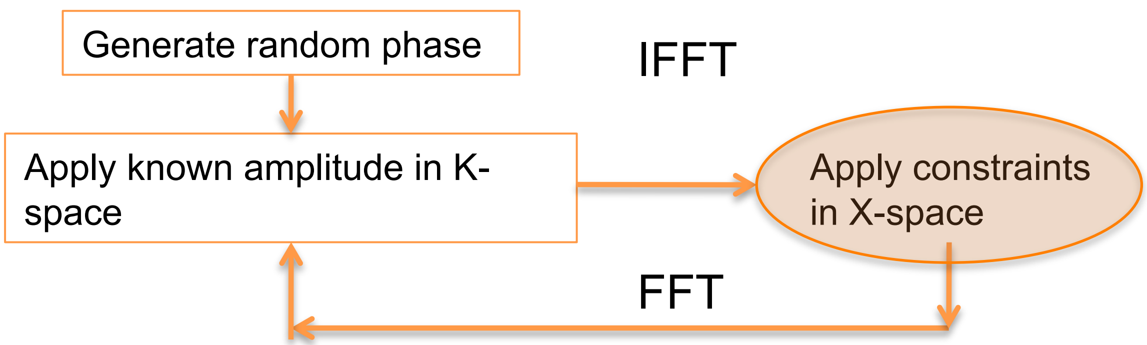
-Support Constraint

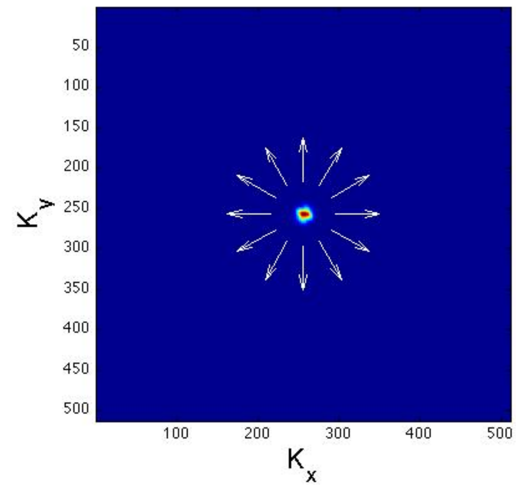
The signal is equal to 0 outside of a finite domain in X. At each iteration this condition is enforced by the algorithm.

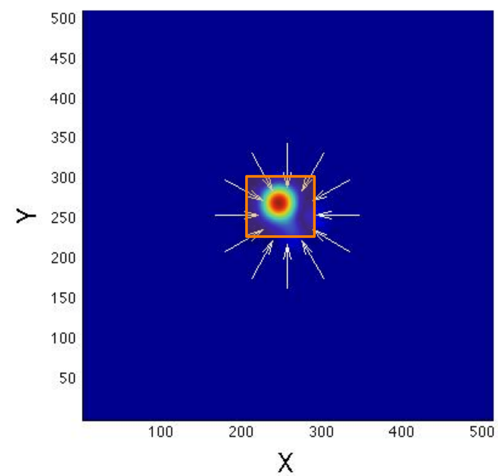
Oversampling condition: $\delta k < 2\pi / L_{sample}$

Finer sampling in K space gives a stronger constraint in X-space.

-Positivity Constraint
Enforce that signal is real and positive in the domain

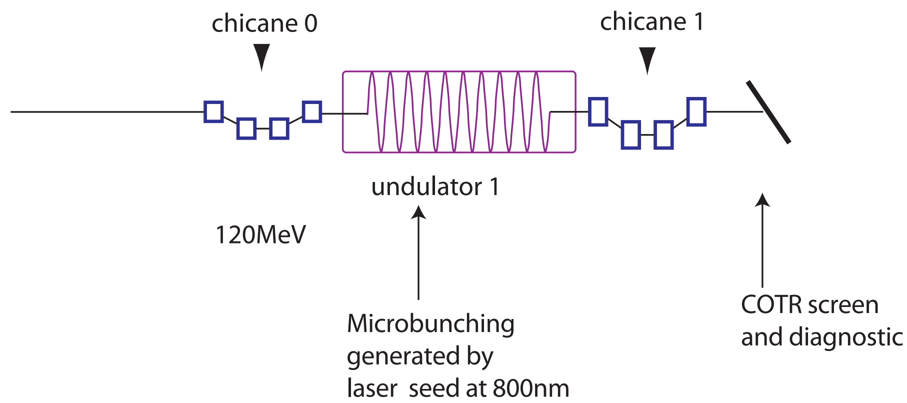


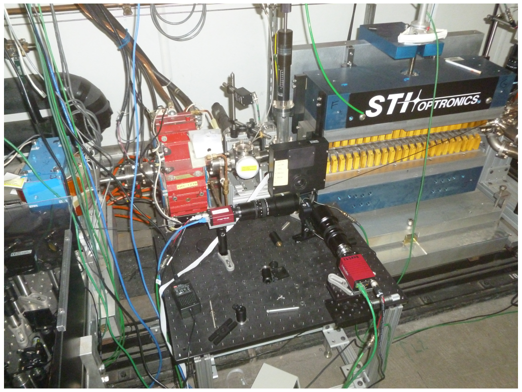




J. R. Fienup, "Phase retrieval algorithms: a comparison," Appl. Opt. 21, 2758-2769 (1982) .
http://www.opticsinfobase.org/abstract.cfm?URI=ao-21-15-2758

Experimental Demonstration at NLCTA



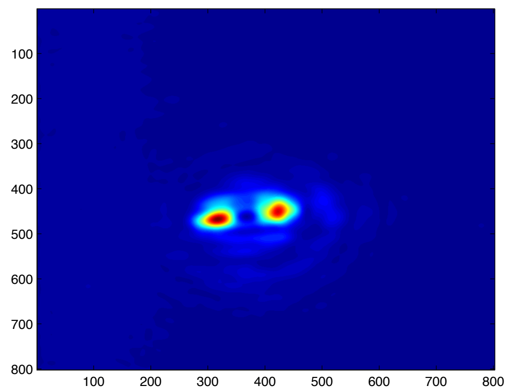


-COTR experiment performed with the ECHO beamline setup.

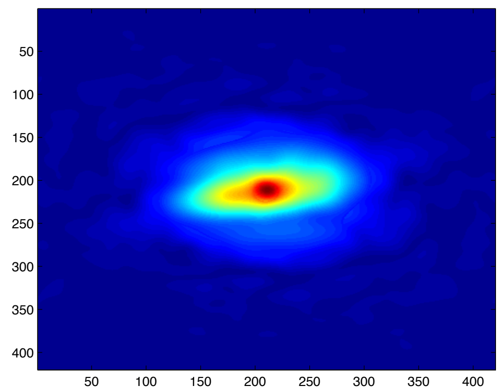
-Proof of principle: transverse optical replica of uncompressed beam.
Reconstruction benchmarked with incoherent OTR!

-Seeded microbunching: positivity constraint in x-y

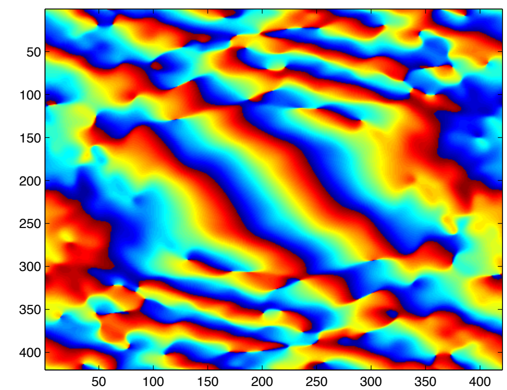
EXPERIMENT AT 800NM



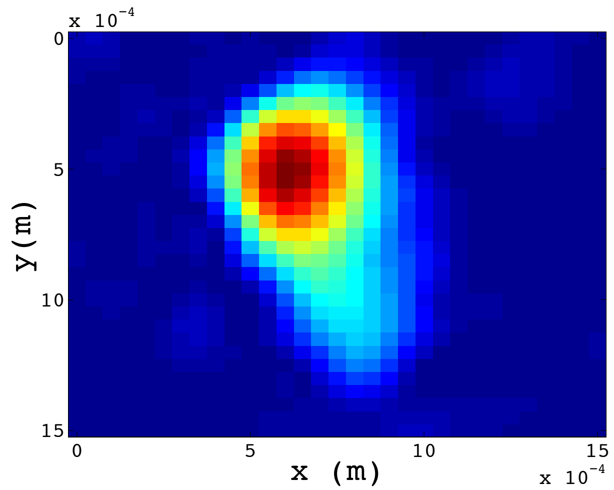
Far Field COTR



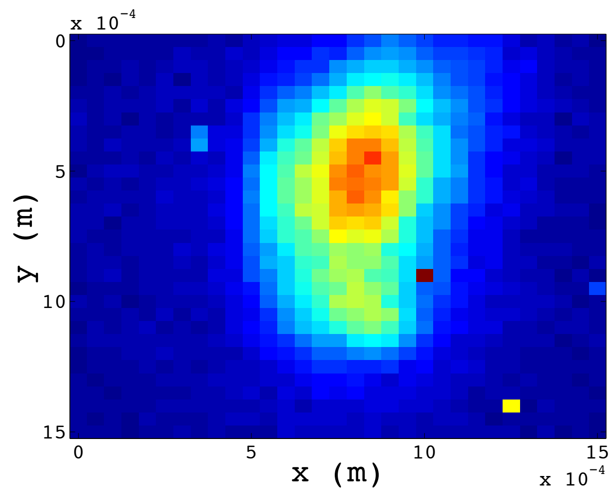
|B (k_x,k_y) |



Phase{B (k_x,k_y) }



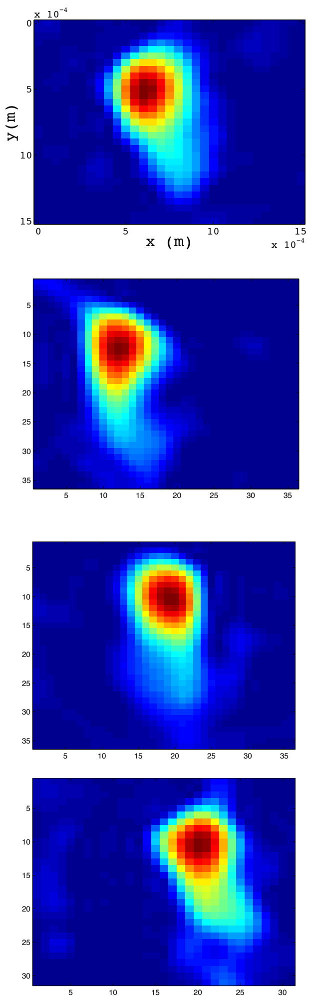
Reconstructed Microbunching



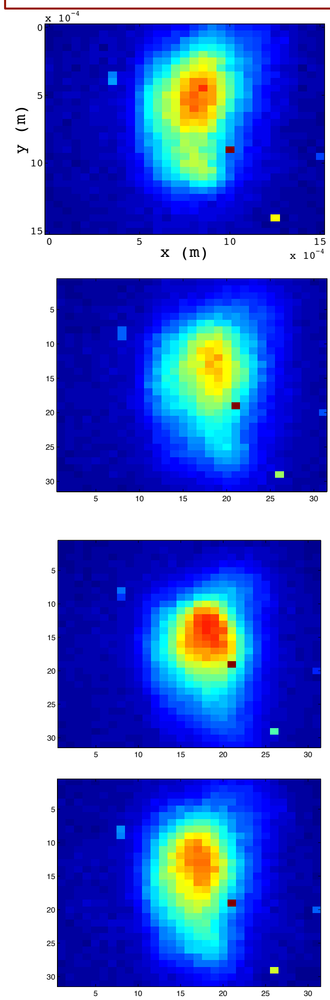
Near Field Incoherent OTR

EXPERIMENT AT 800NM

Reconstruction



Near Field Incoherent OTR



Beam fluctuates from shot-to-shot.

Characteristic size and shape consistent from shot to shot.

(near field and reconstructed images come from different shots!)

Features and Advantages

- Single-shot reconstruction of beam microbunching.

  (a lot of physics can be understood if we extend it to non-positive microbunching…)

- Beam imaging technique robust to microbunching instability and other forms of parasitic microbunching.

- Higher resolution for high energy beams

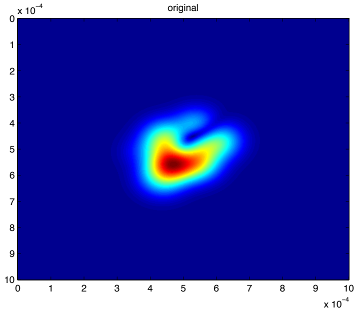
  (OTR resolution ~$\gamma\lambda$ TOR resolution limited by dynamic range of the camera).

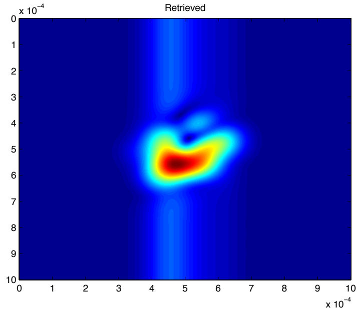
Possible Improvements

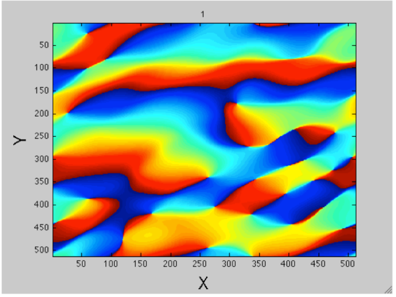
-Use of undulator radiation

(no null on axis, collect more information on the beam's Fourier transform)

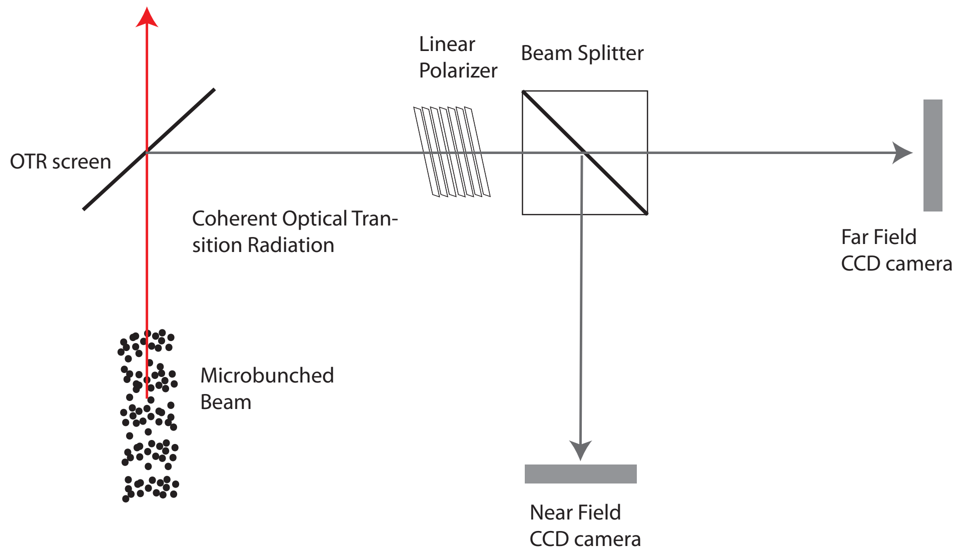
-Simultaneous near/far field COTR imaging for non-positive microbunching (OAM or space-charge)

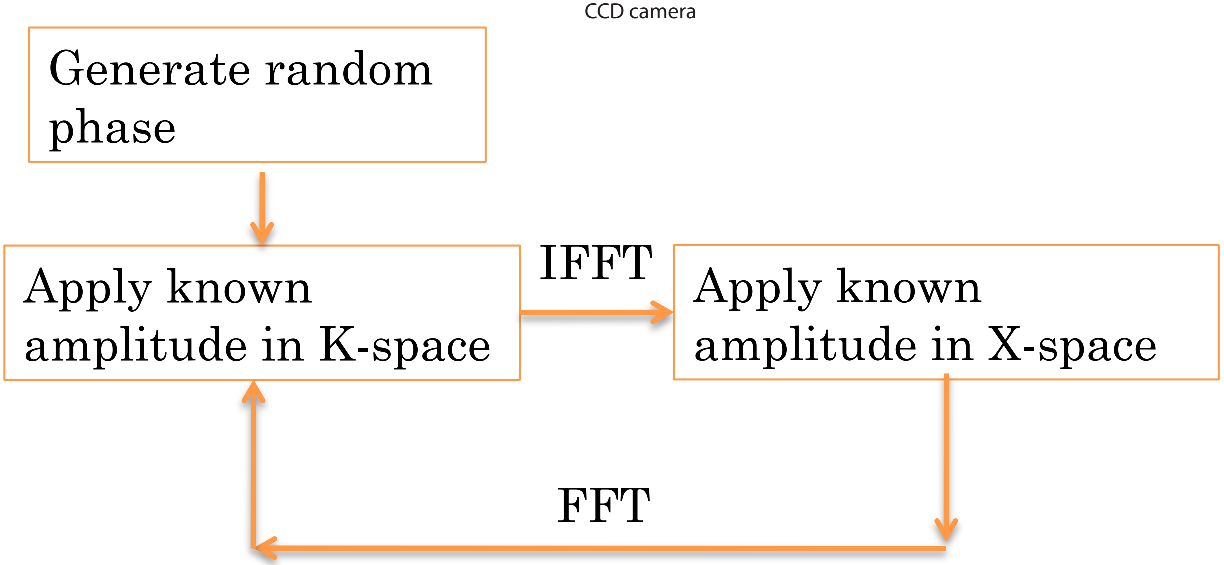






Double Intensity Measurement





In the double intensity measurement the phase retrieval is performed on ONE polarization of the COTR field

The constraint in X space is the measured amplitude in the near field zone

The microbunching distribution is recovered by deconvolving the final signal with the OTR Green's function.

Summary and Conclusions

- We have developed a six-dimensional theory of the space-charge eigenmodes of a beam with finite emittance, betatron motion, energy-spread and 3-D field effects.

  new results:

  -emittance induced Landau damping;

  -coupling of space-charge to betatron motion.

- The theory of space-charge waves has been used to develop a fully kinetic model of the space-charge induced microbunching instability.

  -emittance effects in space-charge amplification/suppression of microbunching;

  -amplification/suppression of higher-order modes

  -design of a seeded longitudinal space-charge amplifier at the NLCTA

- A microbunching reconstruction experiment, based on phase-retrieval techniques, has been designed and performed at the NLCTA test facility at SLAC.

  -successful reconstruction of seeded microbunching with application to compressed beams

  -double intensity measurement for more exotic microbunching structures?

Acknowledgments

The author would like to acknowledge:

J. Rosenzweig, C. Pellegrini, E. Hemsing, P. Musumeci and G. Morales

for useful discussions and suggestions.

Many many thanks to the NLCTA team at SLAC for their support in the COTR experiment…

(Mike, Stephen, Dao, Doug, Janice, Keith, Erik, Carsten and Thor)

And thanks to you all for inviting me here!