First Demonstration of Optical Frequency Shot-Noise Suppression in Relativistic Electron-beams

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Outline

1. Physical origin of current noise suppression

2. Experimental demonstration of current noise suppression – beating the Shot-Noise limit

3. 1-D and 3-D theory of current noise suppression

4. Model interpretation of the experiment

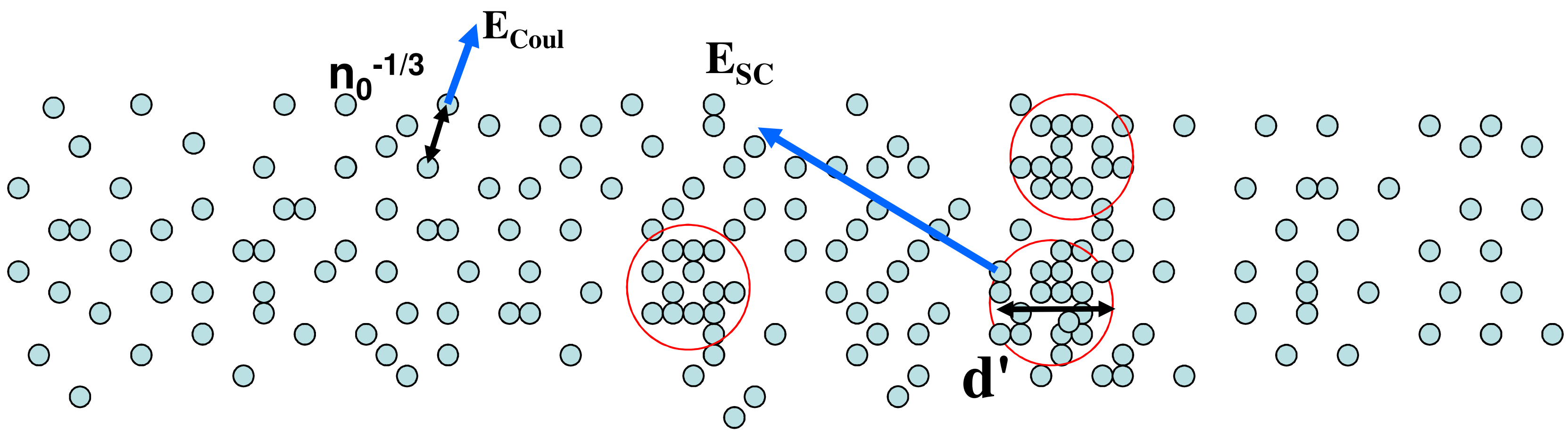
5. Application of Noise suppression to enhancement of FEL coherence

6. Limits of current-noise and SASE suppression

Physics of Collective Micro-Dynamics in a Charged Particle Beam:

- Spatially coherent Coulomb interaction micro-dynamics.

- Effect of particle self-ordering and current shot-noise suppression at optical frequencies.



3-D Homogenization Trend

A simple physical argument:

Inter-particle Coulomb force: $\varepsilon_{Coul} = e^2 / 4\pi\varepsilon_0 n'^{-1/3}_0$

Space-charge force: $\varepsilon_{sc} = e^2 \Delta N' / 2\pi\varepsilon_0 d'$

Poisson statistics: $\Delta N' = N'^{1/2}$ $N' = \left(\pi d'^3 n'_0 / 6\right)^{1/2}$

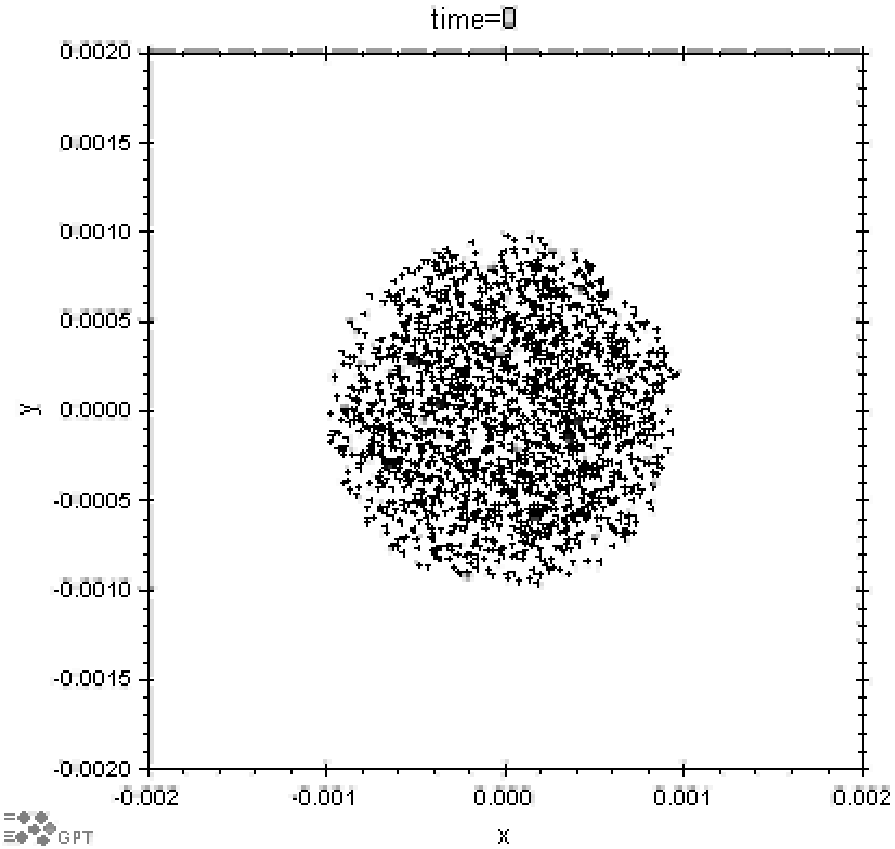
When $\boldsymbol{\varepsilon_{sc} > \varepsilon_{coul}}$?

$$\frac{\varepsilon_{sc}}{\varepsilon_{Coul}} = \left(\frac{2\pi}{3} \frac{d'}{n'^{-1/3}_0} \right)^{1/2} \rangle 1$$

<u>Answer</u>: **$d' > n_0^{-1/3}$**

<u>Note</u>: Process leads to velocity spread growth

Expansion of a Sphere Shaped Bunch of Uniformly Distributed Charges in Time Period $t = \pi/2\omega_p$



t=0[Sec], R=1[mm]

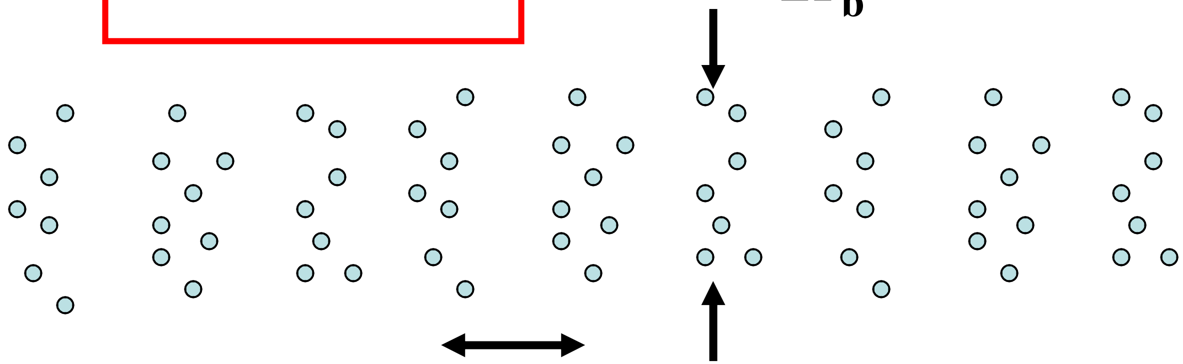
Conditions for a Charged-Particle Beam to Exhibit Spatially-Coherent Current Shot-Noise Suppression:

$$\left.\overline{\left|\breve{i}(\omega)\right|^2}\right|_{t\cong t_{\pi/2}} \langle \quad \left.\overline{\left|\breve{i}(\omega)\right|^2}\right|_{t=0} = eI_b$$

1. **Cold beam: current shot-noise dominated (non-equilibrium plasma !)**
2. **Longitudinal interaction (single Langumir mode)**

$$\lambda' > 2r_b$$

2r$_b$



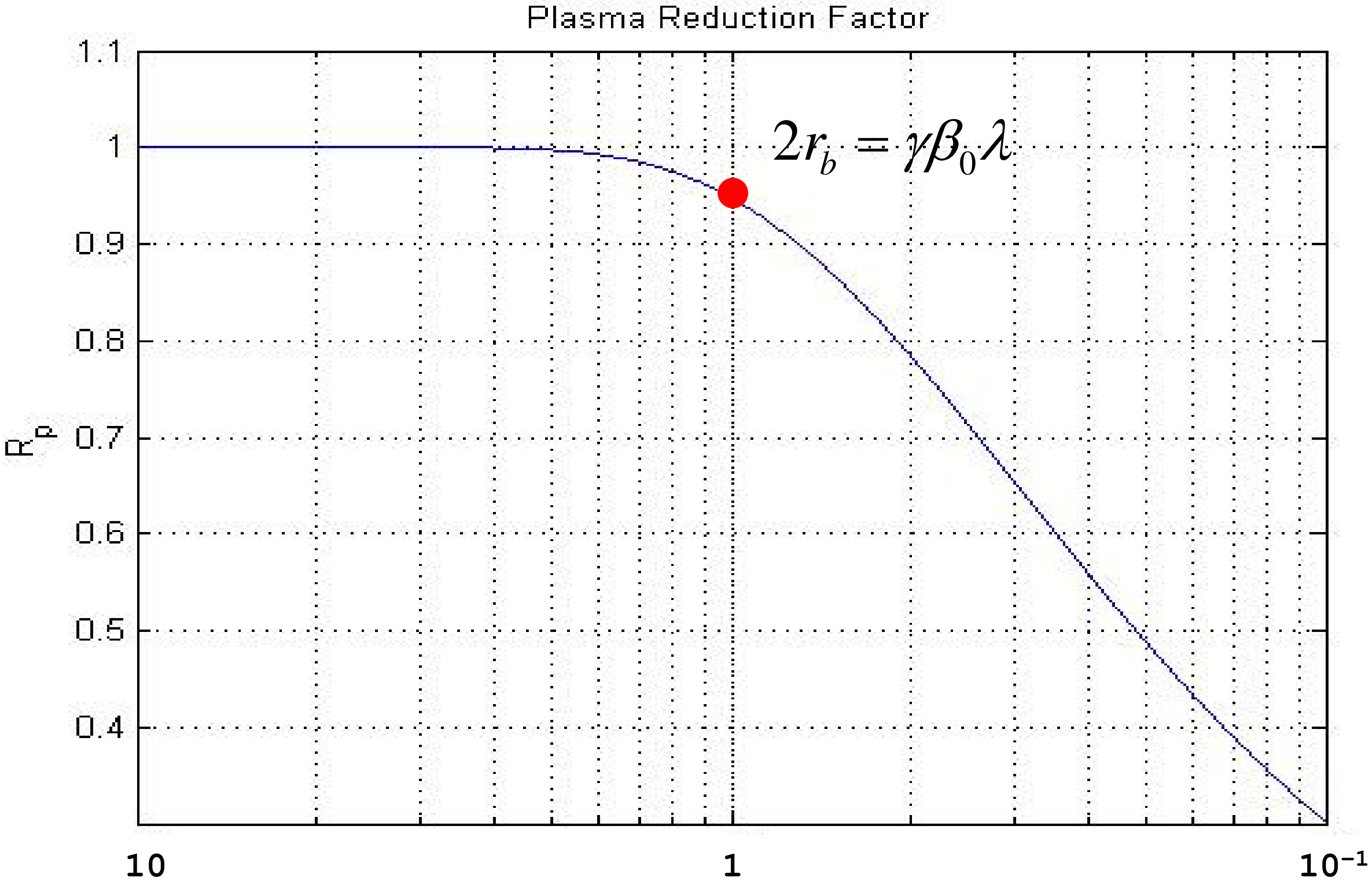
$$\lambda' = \lambda\gamma\beta$$

(Beam rest frame picture)

Plasma Reduction Factor – Fundamental Langmuir Plasma Wave Mode

$$r_p = \{1 - \pi u K_1(\pi u)\}^{1/2}$$

[M. Venturini, NIM-PR-A 599 (2009) 140]



$$u = \frac{2r_b}{\gamma\beta\lambda}$$

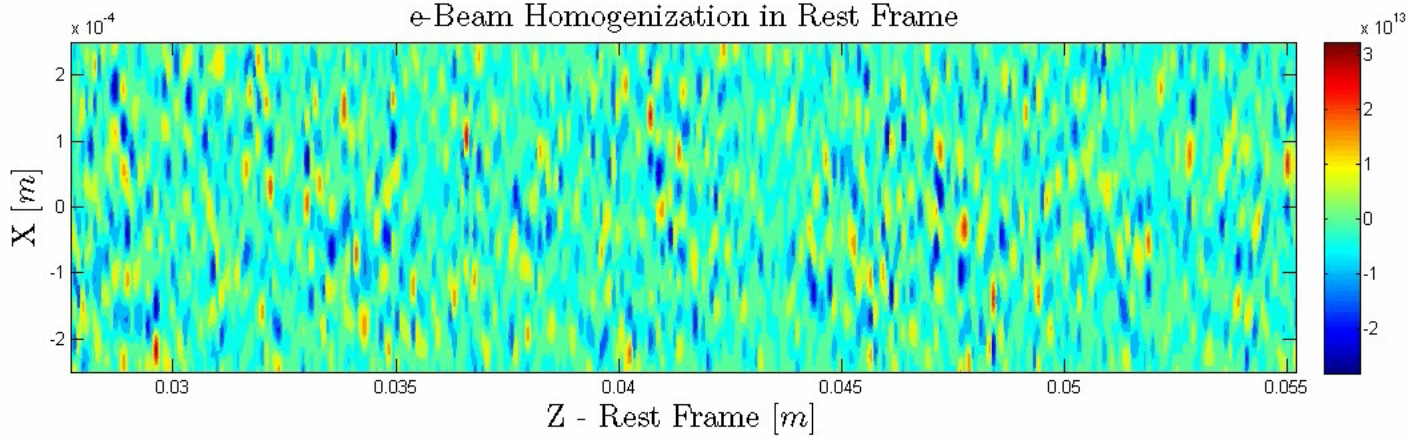
3-D Numerical Simulations

A. Nause, E. Dyunin, A. Gover,
JAP 107, 103101 (2010).

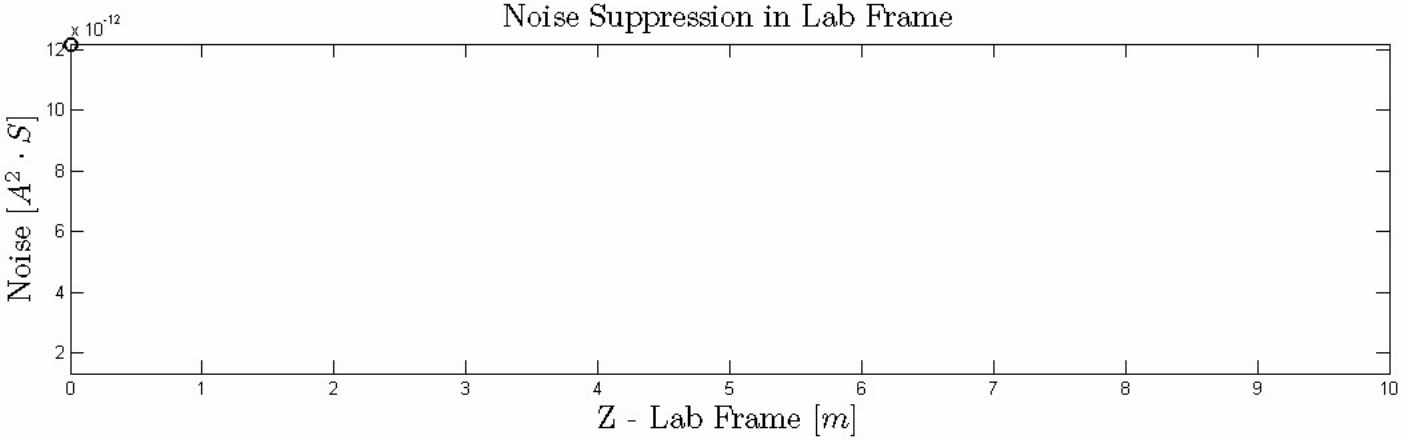
100,000 particles over 2 pS duration to increase resolution,
70 MeV energy, 200pC charge

HOMOGENIZATION (λ=5-10 μm)

Density
In beam
frame

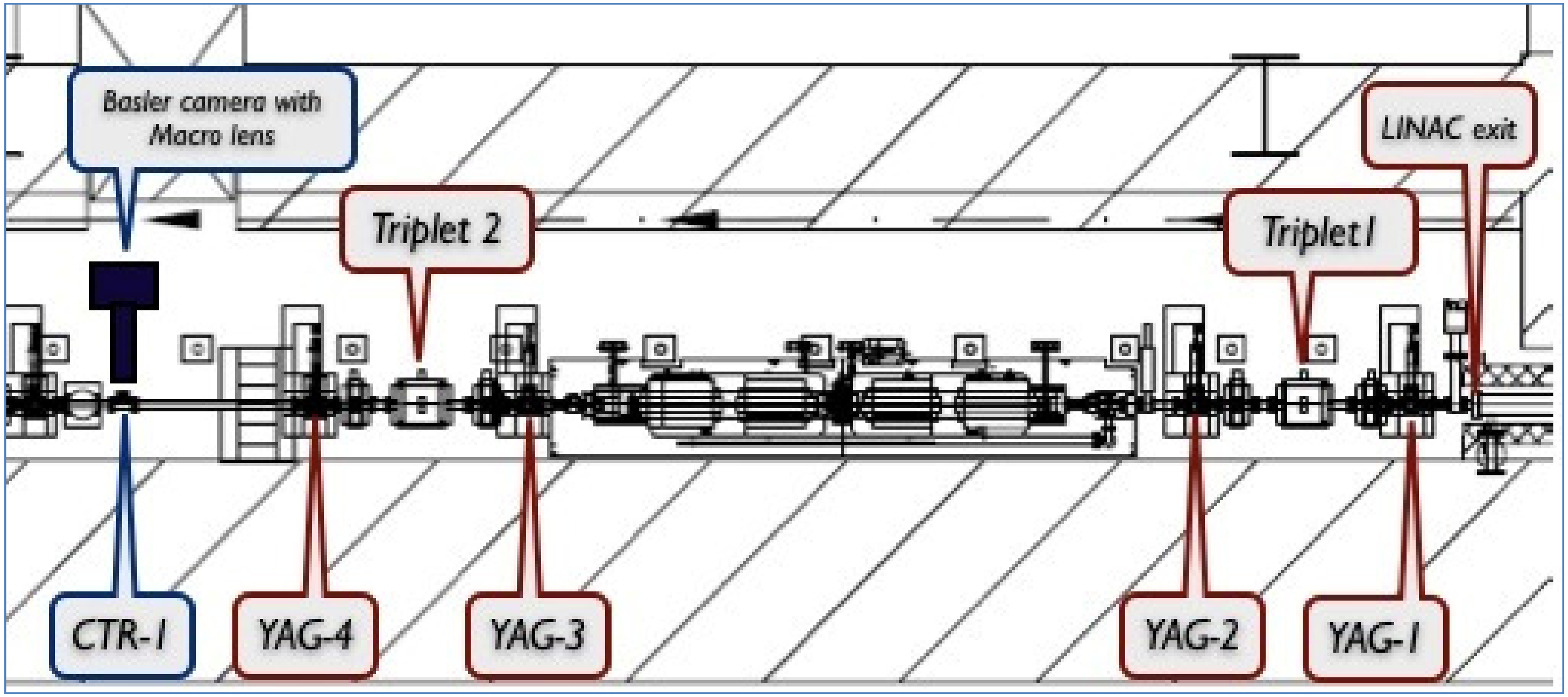


$$\frac{\overline{\left|\breve{I}(z,\omega)\right|^2}}{\overline{\left|\breve{I}(0,\omega)\right|^2}}$$

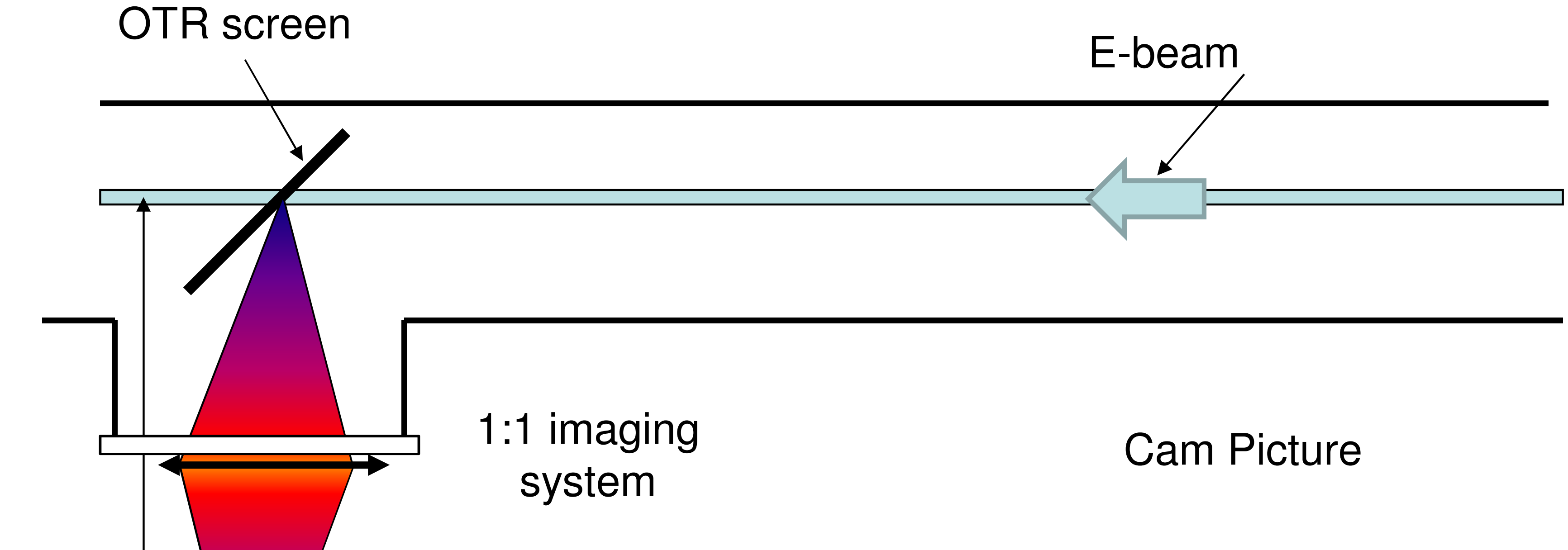


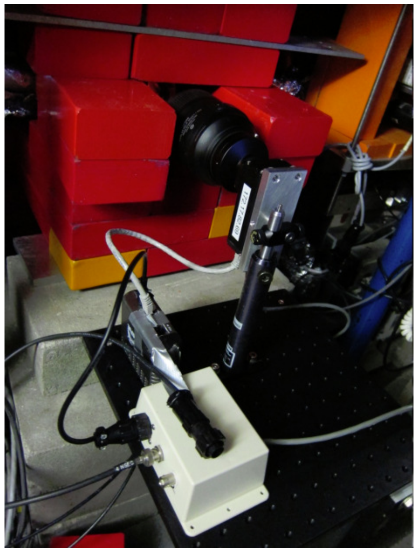
NOISE SUPPRESSION EXPERIMENT IN ATF
OCTOBER 2011

EXPERIMENTAL SETUP

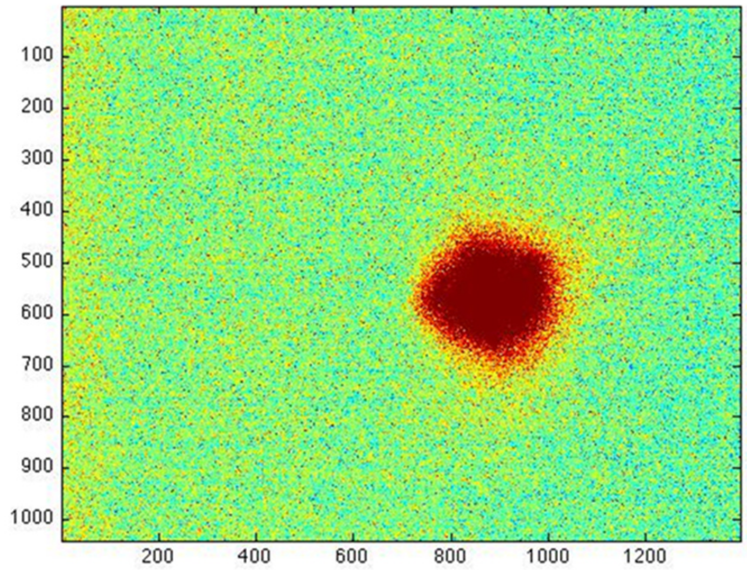


Experimental Setup



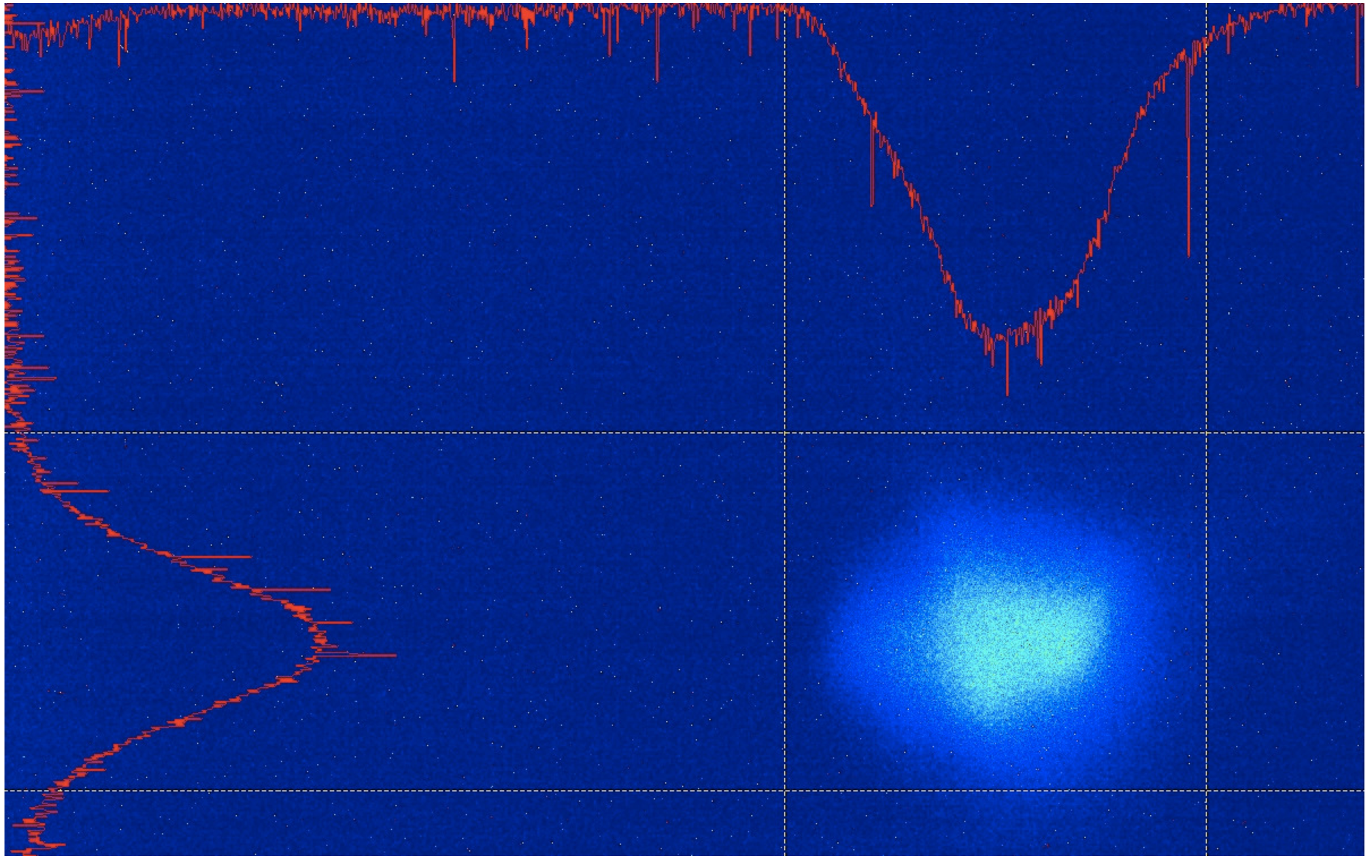


Cam Picture



OTR-beam profile

(expanded dynamic range of the frame grabber record M. Fedurin - ATF)



Operating Parameters

Pulse length: 5 ps

Beam energy: 50 – 70 MeV

Beam current: 40-100 A

Emittance: ~3 mm-mrad

Initial beam size: 400-500 μm

Convergence: ~2 mrad

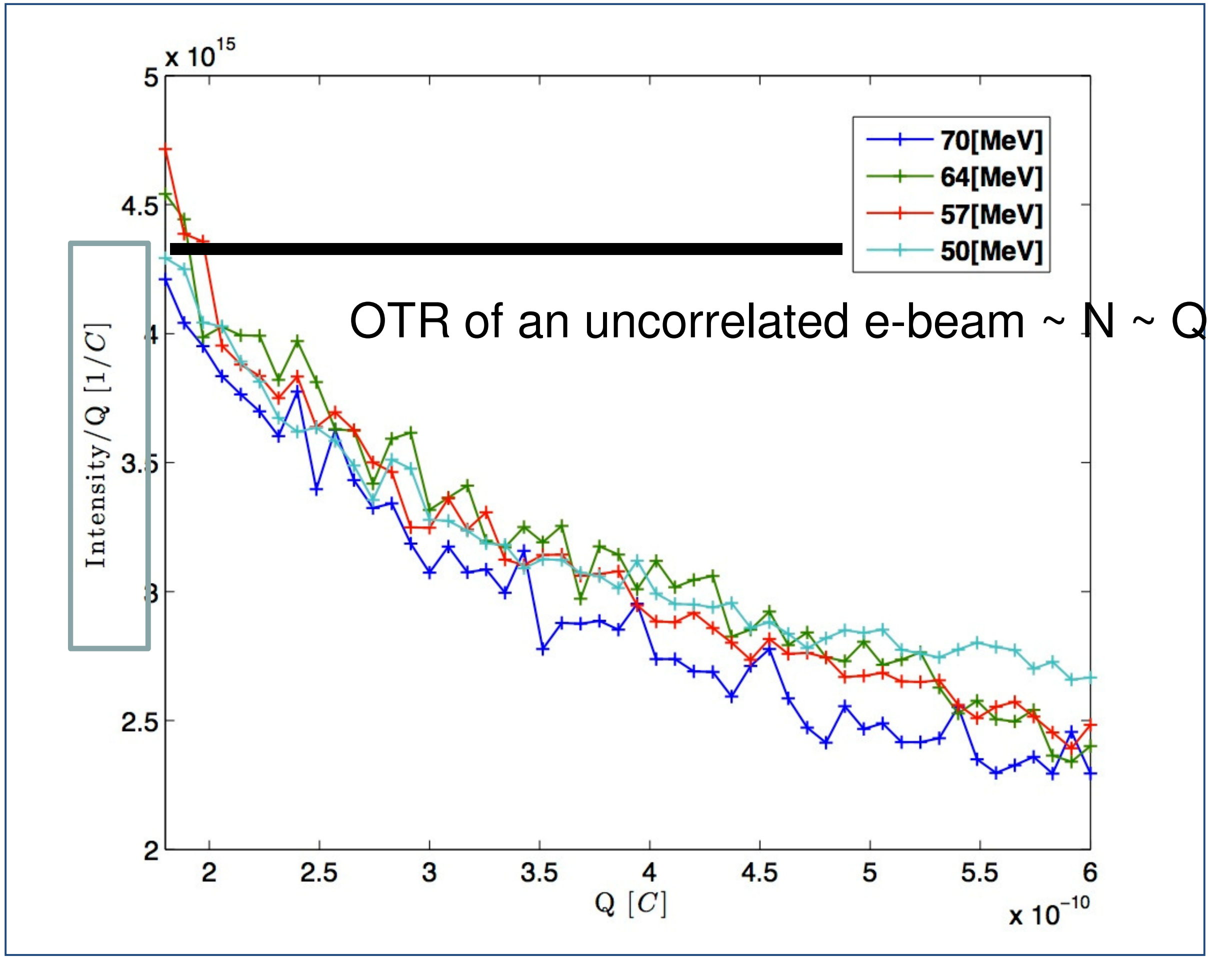
Acceleration phase: on crest

Copper OTR screen

Basler CCD camera equipped with a Nikkor macro lens (100 mm)

Cam sensitivity: 0.4 – 1 μm

Experimental Results



ANALYTICAL FLUID-PLASMA LINEAR MODEL

[H. Haus and F. N. H. Robinson, Proc. IRE 43, 981 (1955)]

[A. Gover, E. Dyunin, PRL 102, 154801 (2009)]

Coherent Plasma Oscillation in an e-Beam Drift Section

$$\breve{i}(L_d,\omega)=\left[\breve{i}(0,\omega)\cos\phi_p - i\breve{V}(0,\omega)\left(\sin\phi_p / W_d\right)\right]e^{i\phi_b(L_d)}$$

$$\breve{V}(L_d,\omega)=\left[-i\breve{i}(0,\omega)W_d\sin\phi_p + \breve{V}(0,\omega)\cos\phi_p\right]e^{i\phi_b(L_d)}$$

$$\breve{V}(z,\omega)=-\left(mc^2/e\right)\breve{\gamma}(z,\omega)=-\left(mc^2/e\right)\gamma_0^3 v_0 \breve{v}(\omega)$$

(Chu's Relativistic Kinetic Voltage)

$$\phi_b=\frac{\omega}{v_z}L_d \qquad \phi_p=\theta_{pr}L_d \qquad \theta_{pr}=r_p\frac{\omega_p'}{v_0}$$

$$\omega_p'=\left(\frac{e^2 n_0}{m\varepsilon_0\gamma^3}\right)^{1/2} \qquad W_d=r_p^2\sqrt{\mu_0/\varepsilon_0}\Big/k\theta_{prd}A_e$$

At the Cathode (no-correlation point)

Current and velocity noise – uncorrelated:

$$\overline{\left|\breve{i}(\omega)\right|^2} = \frac{1}{T}\left\langle\left|\breve{i}(\omega)\right|^2\right\rangle_{N_T} = eI_b$$

$$\overline{\left|\breve{V}(\omega)\right|^2} = \frac{1}{T}\left\langle\left|\breve{V}(\omega)\right|^2\right\rangle_{N_T} = \frac{(\delta E_c)^2}{eI_b}$$

$$\left(\overline{\left|\breve{I}(\omega)\right|^2}\right)^{1/2}\left(\overline{\left|\breve{V}(\omega)\right|^2}\right)^{1/2} = \delta E_c$$

Coherent Plasma Oscillation in an e-Beam Drift Section

$$\breve{i}\left(L_d,\omega\right)=\left[\breve{i}\left(0,\omega\right)\cos\phi_p-i\breve{V}\left(0,\omega\right)\left(\sin\phi_p/W_d\right)\right]e^{i\phi_b(L_d)}$$

$$N^2=\frac{\overline{\left|\breve{V}(0,\omega)\right|^2}}{W_d^2\overline{\left|\breve{i}(0,\omega)\right|^2}}=\left(\frac{\lambda_D}{\lambda}\right)^2 \qquad k_D=\frac{2\pi}{\lambda_D}=\frac{\omega_{pl}}{\delta v}$$

$$gain=\frac{\overline{\left|\breve{i}(L_d,\omega)\right|^2}}{\overline{\left|\breve{i}(0,\omega)\right|^2}}=\cos^2\phi_p+N^2\sin^2\phi_p$$

$$\phi_p=\theta_{pr}L_d \qquad \theta_{pr}=r_p\frac{\omega_p'}{v_0} \qquad \omega_p'=\left(\frac{e^2n_0}{m\varepsilon_0\gamma^3}\right)^{1/2}$$

LABORATORY CONTROL OVER COLLECTIVE MICRODYNAMICS AND NOISE SUPPRESSION:

FREE DRIFT FOCUSING

BEAM WAIST NOISE SUPPRESSION THEOREM

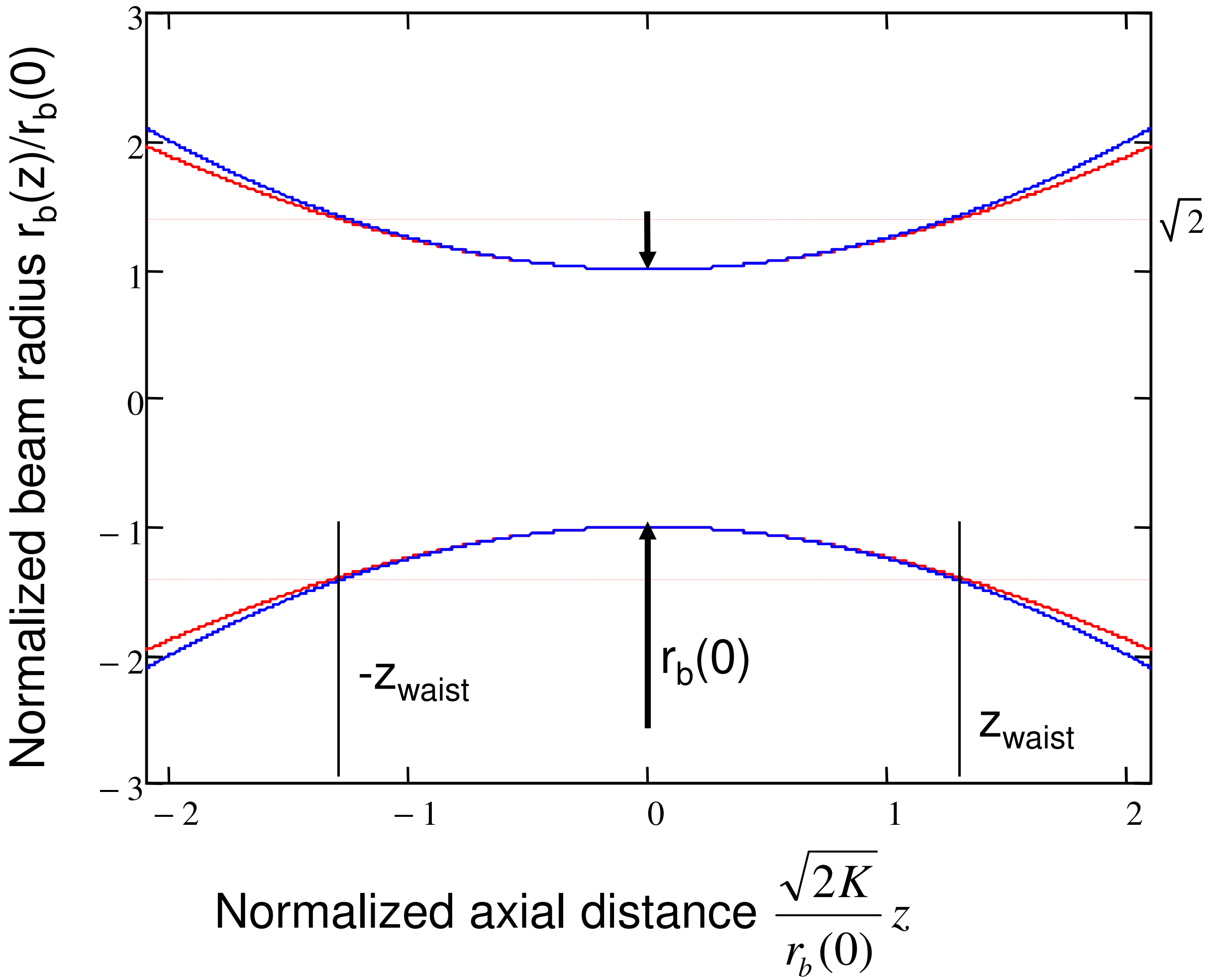
[A. Gover, E. Dyunin, PRL 102, 154801 (2009) - Appendix]

- A sufficient condition for quarter plasma wavelength oscillation is space-charge dominated transport of a beam through a waist.
- This is also the condition for maximum noise suppression, if the beam noise is initially uncorrelated and dominated by current shot-noise.

Waist Definition

Space Charge Dominated Regime (ε = 0)

Numerical solution — Analytic approximation: $r_b(z) = r_0\left(1 + \frac{K}{2}\left(\frac{z}{r_0}\right)^2\right)$



Plasma Phase Accumulation in a Waist

[Space Charge Dominated Regime (ε = 0)]

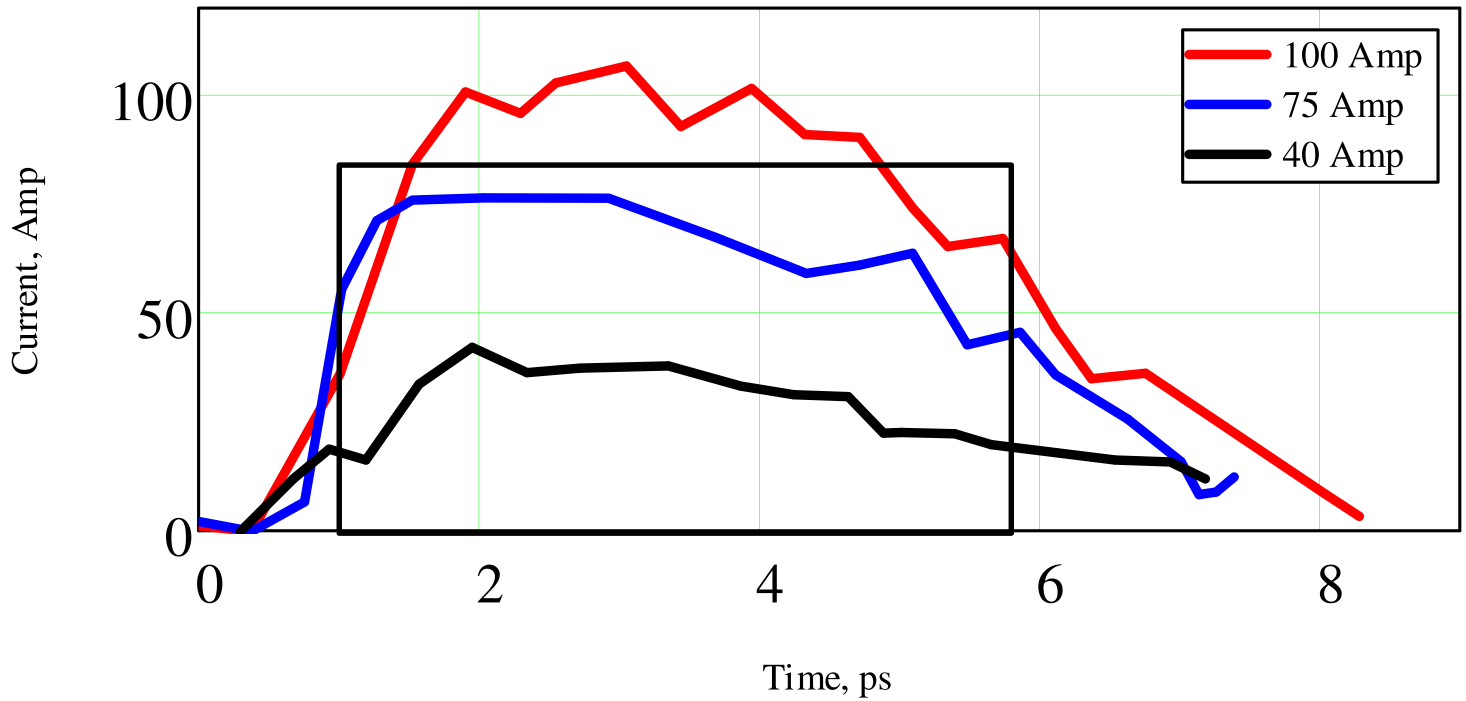
$$\varphi_{p_{waist}} = \int_{-z_{waist}}^{z_{waist}} \theta_p(z)dz = \int_{-z_{waist}}^{z_{waist}} \sqrt{\frac{I_0 Z_0}{\pi r_b^2(z)\frac{mc^2}{e}\beta_0^3\gamma_0^3}}dz$$

$$r_b(z) = r_0\left(1 + \frac{K}{2}\left(\frac{z}{r_0}\right)^2\right) \qquad z_{waist} = r_0\sqrt{\frac{2(\sqrt{2}-1)}{K}}$$

$$K = \frac{2I_0}{I_A(\gamma\beta)^3}$$ relativistic perveance

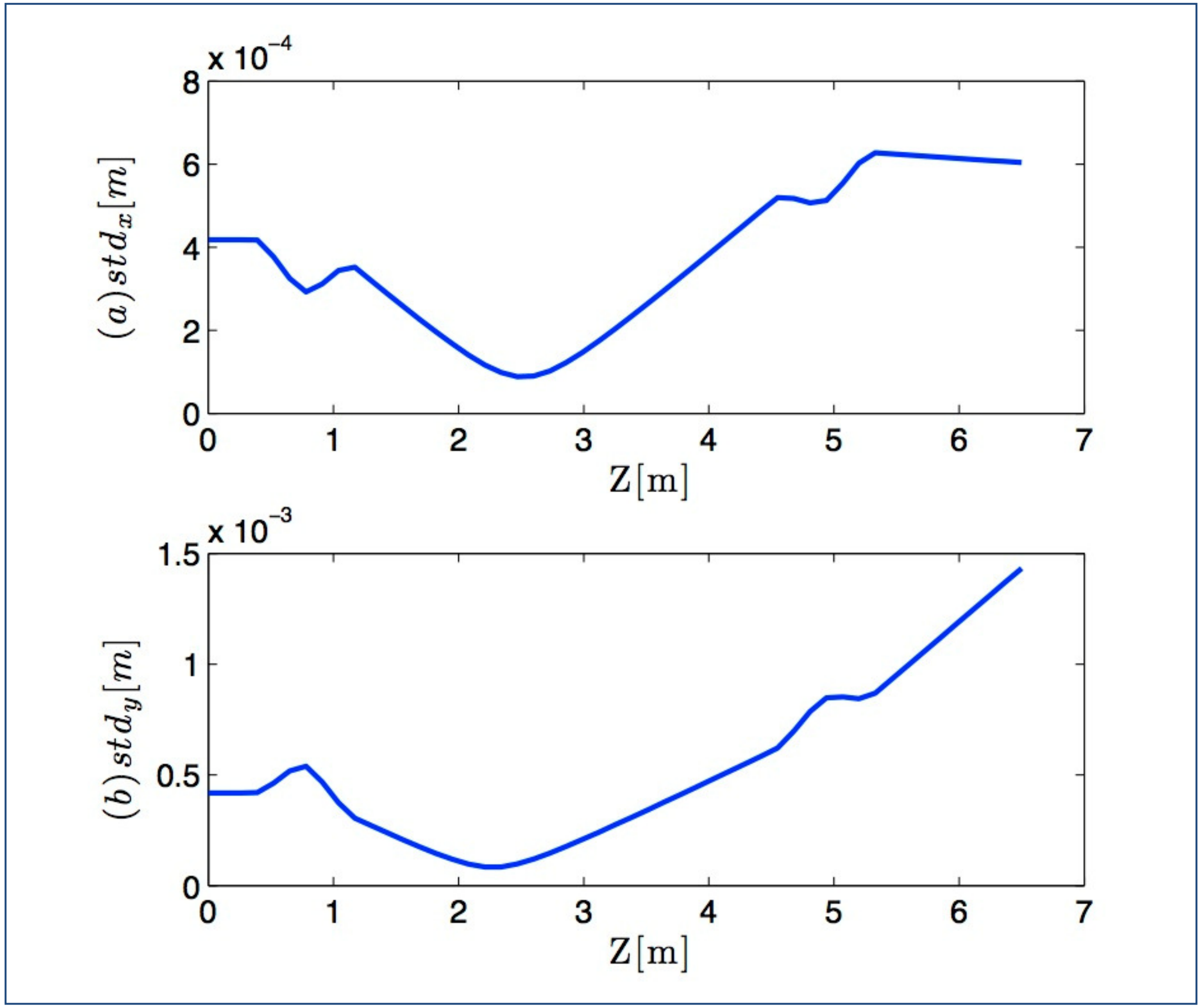
$$\varphi_{p_{waist}}^{sch} = \int_{-z_{waist}}^{z_{waist}} \frac{1}{(2r_0^2 + \frac{2I_0}{I_A\beta_0^3\gamma_0^3}z^2)}\sqrt{\frac{4r_0^2 I_0 Z_0}{\pi\frac{mc^2}{e}\beta_0^3\gamma_0^3}}dz = 4\,\mathrm{atan}\left(\sqrt{(\sqrt{2}-1)}\right) = \frac{\pi}{2}$$

MODEL INTERPRETATION OF THE EXPERIMENT



100 Amp
75 Amp
40 Amp
100
50
0
0
2
4
6
8
Current, Amp
Time, ps

Beam Profile Along Trajectory (GPT)



Collective interaction – z varying parameters

$$\varphi_p(z) = \int_0^z \theta_{pr}(z')dz' \qquad \theta_p^2(z) = \frac{eZ_0 I_0}{mc^2 A_e(z)\gamma_0^3 \beta_0^3}$$

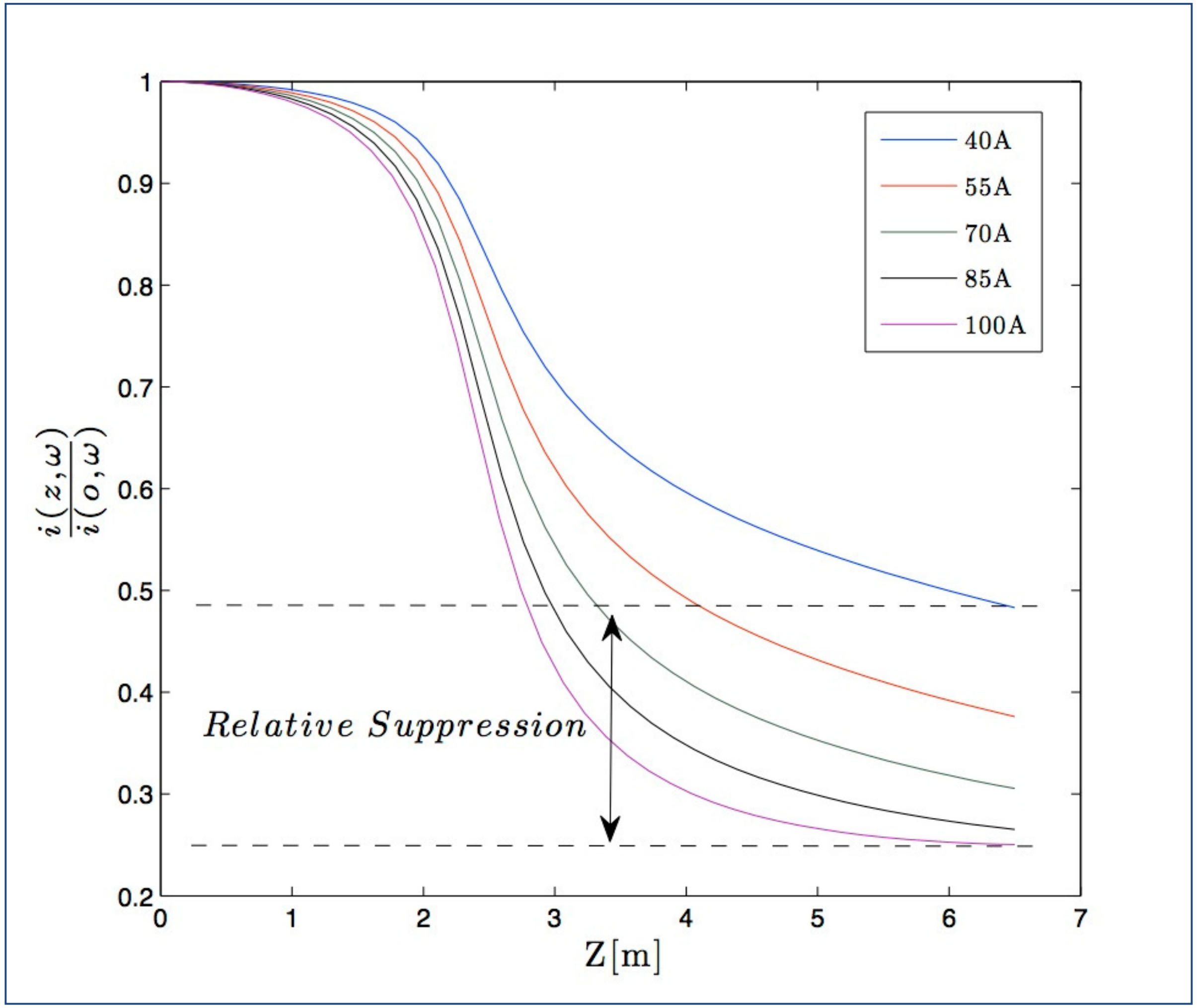
$$\frac{d}{d\varphi_p}\breve{i}(\omega, z) = -\frac{i}{W(z)}\breve{v}(\omega, z)$$

$$\frac{d}{d\varphi_p}\breve{v}(\omega, z) = -iW(z)\breve{i}(\omega, z)$$

$$\frac{d\tilde{i}(z,\omega)}{dz} = -i\omega\varepsilon_0 A_e(z)\theta_p^2(z)\tilde{v}(z,\omega)$$

$$\frac{d\tilde{v}(z,\omega)}{dz} = -\frac{ir_p^2}{\omega\varepsilon_0 A_e(z)}\tilde{i}(z,\omega)$$

COMPUTATION OF NOISE SUPPRESSION WITH BEAM ANGULAR SPREAD

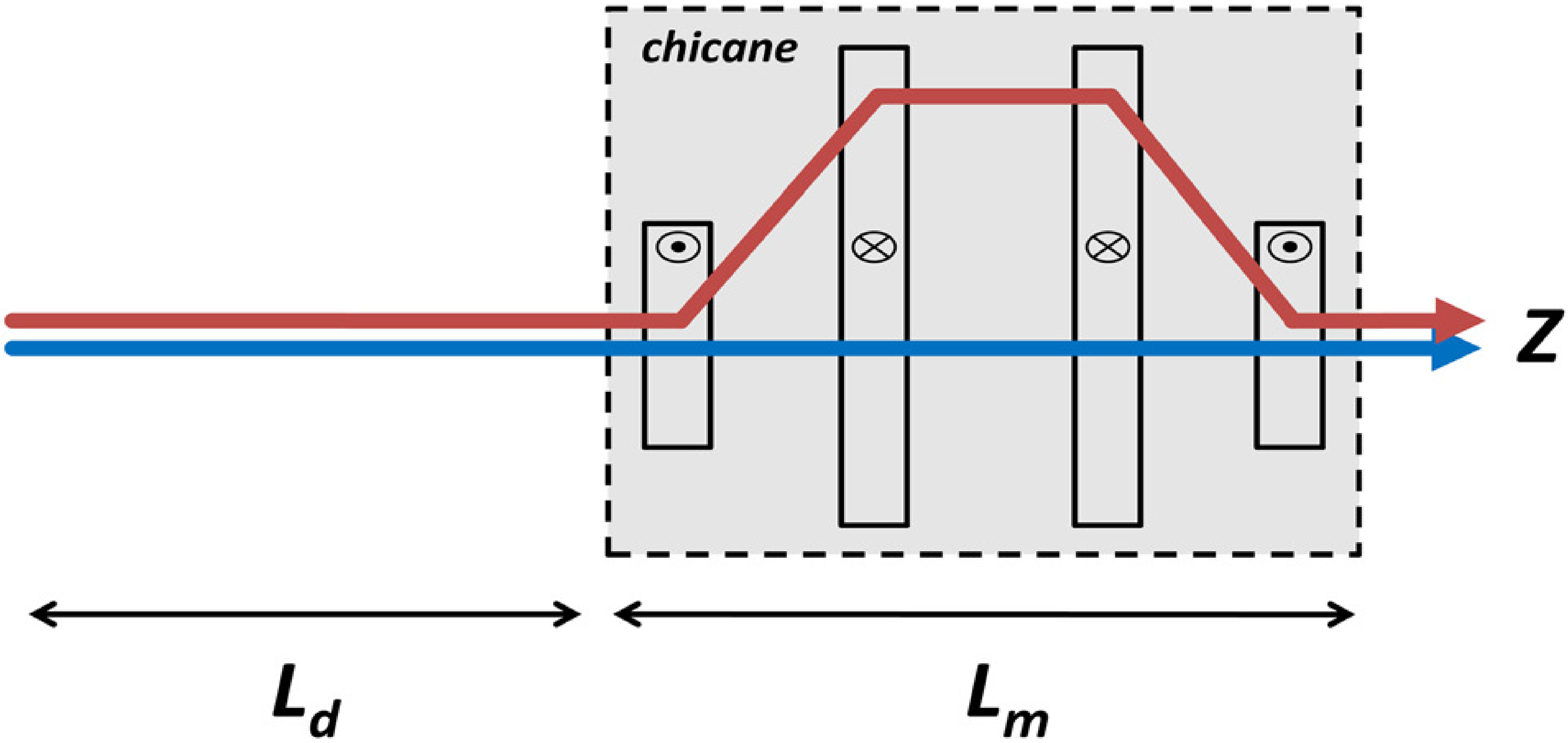


LABORATORY CONTROL OVER COLLECTIVE MICRODYNAMICS AND NOISE SUPPRESSION: **DISPERSIVE TRANSPORT CONTROL**

D. Rathner Z. Huang G. Stupakov, PhysRevSTAB, **14**, 060710 (2011)

A.Gover, E.Dyunin, T.Duchovni, A.Nause, *Phys. of Plasmas*, **18,** 123102 (2011).

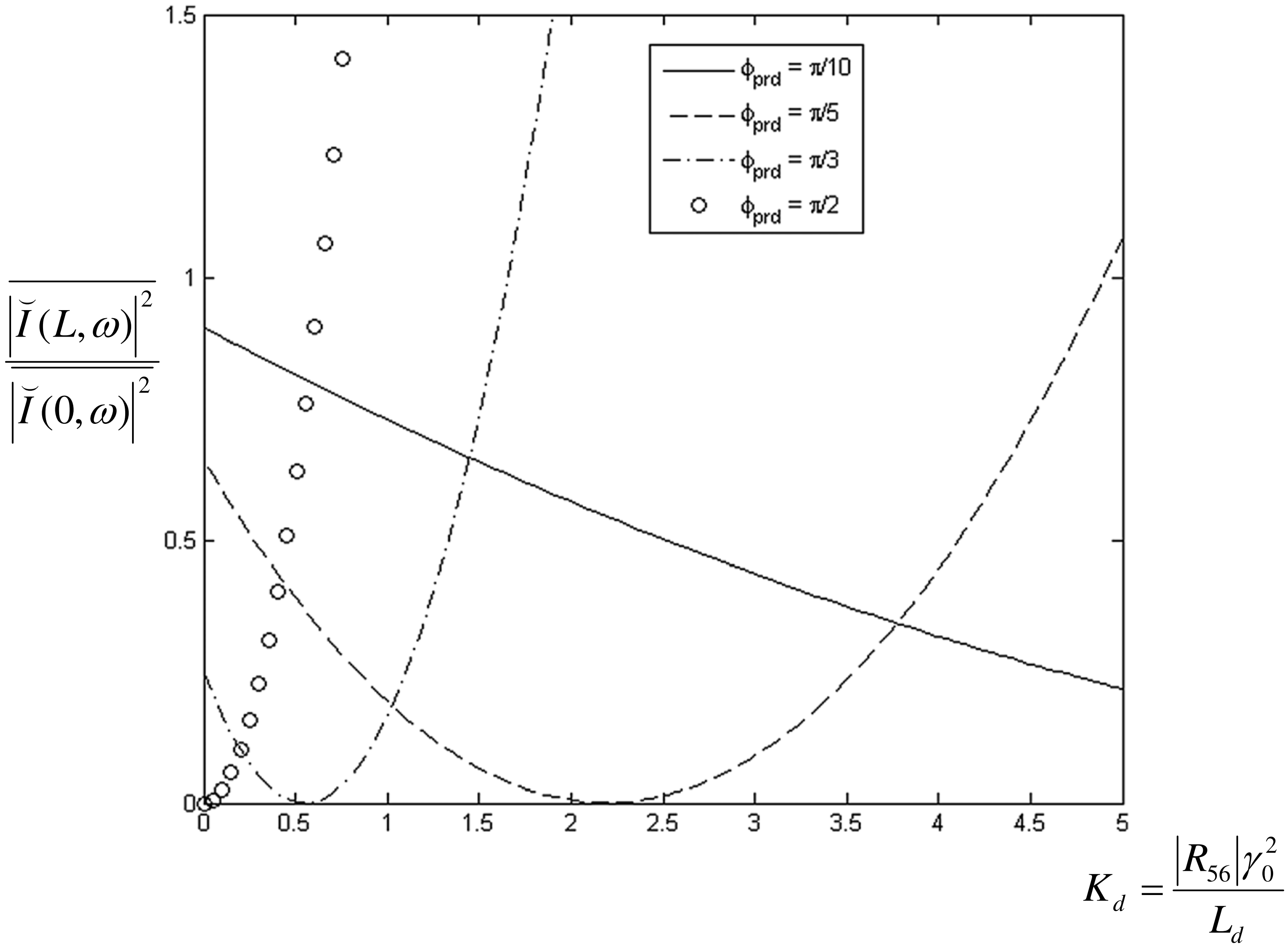
DRIFT/DISPERSION TRANSPORT



Drift

Dispersive Section

Control over current noise from suppression to gain with adjustment of drift length and R_{56}



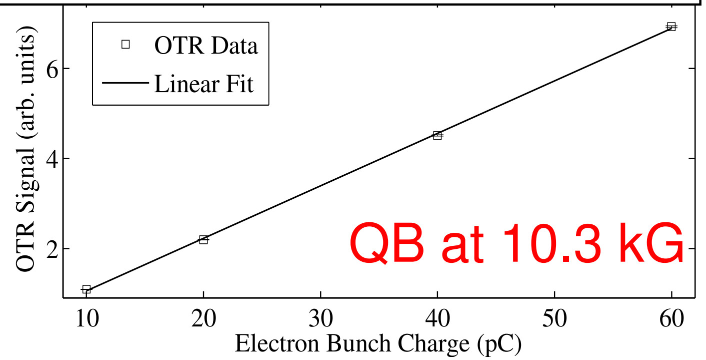
Dispersive Shot Noise Supp.

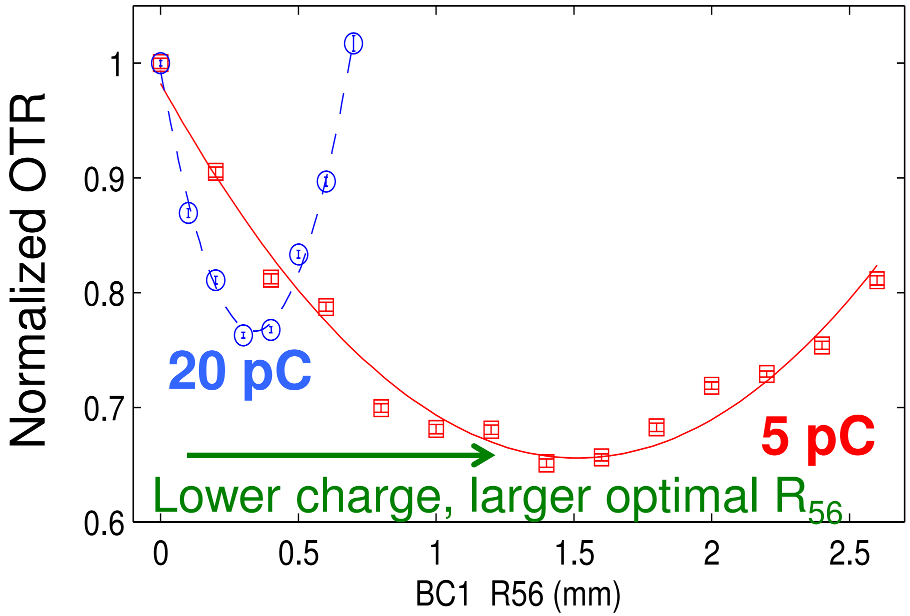
1D Dispersive Shot Noise Suppression

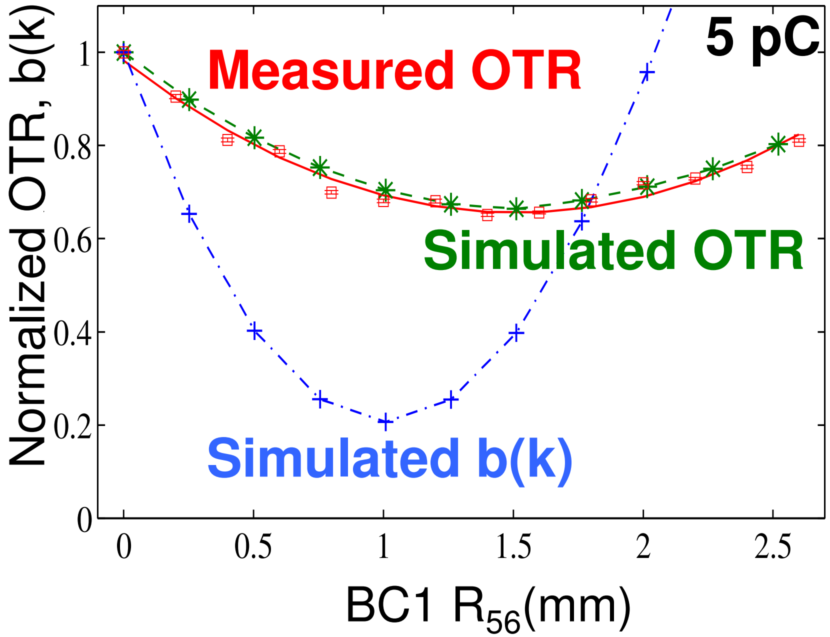
$$N\left\langle |b(k)|^2 \right\rangle = (1 - \Upsilon)^2$$

$$\Upsilon \equiv n_0 R_{56} A$$

OTR proportional to charge at start







SUB-RADIANCE

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

R. H. DICKE
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received August 25, 1953)

IN the usual treatment of spontaneous radiation by a gas, the radiation process is calculated as though the separate molecules radiate independently of each other. To justify this assumption it might be argued that, as a result of the large distance between molecules and subsequent weak interactions, the probability of a given molecule emitting a photon should be independent of the states of other molecules.

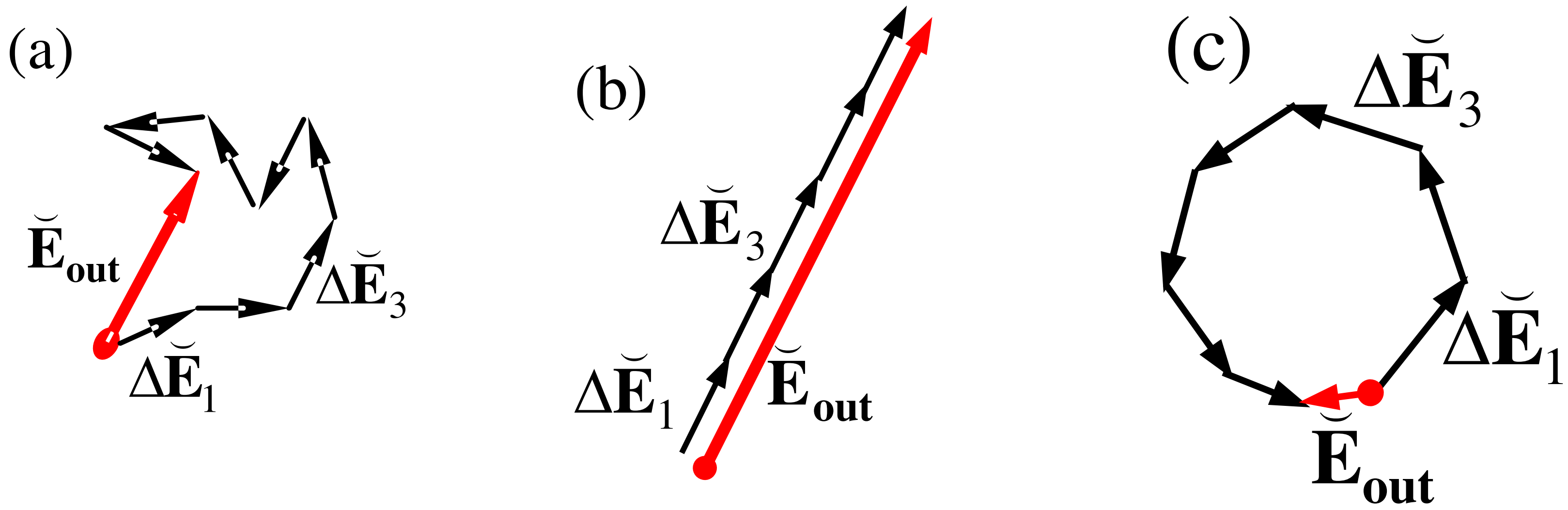
This simplified picture overlooks the fact that all the molecules are interacting with a common radiation field and hence cannot be treated as independent. The model is wrong in principle and many of the results obtained from it are incorrect.

$dP_{in}/d\omega \propto N$ **Spontaneous emission (radiation noise)**

$dP_{in}/d\omega \rangle \alpha N \quad \rightarrow \quad N^2$ **Super-radiance (coherent emission)**

$dP_{in}/d\omega \langle \alpha N \quad \rightarrow \quad 0$ **Sub-radiance**

Fundamental processes of uncorrelated and correlated spontaneous radiative emission in complex C_q plane

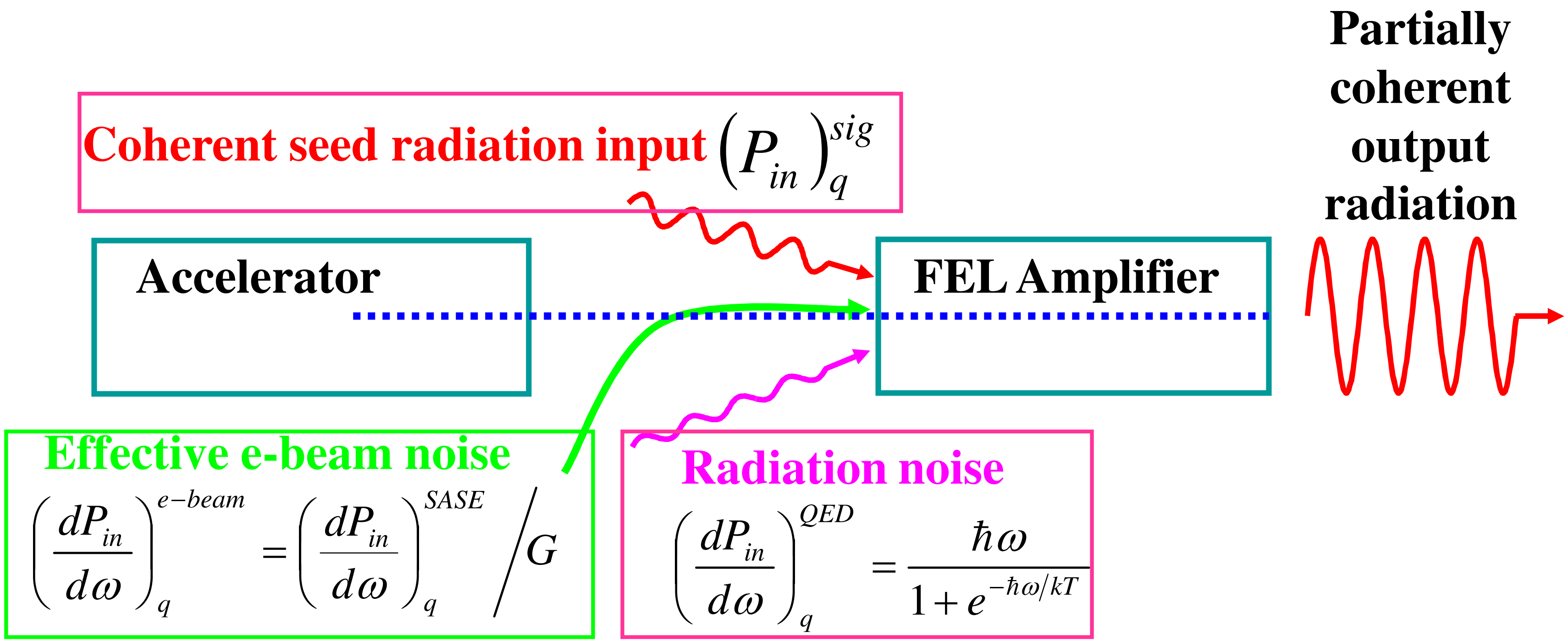


spontaneous emission *superradiance* *subradiance*

SASE SUPPRESSION

Gover, A. Dyunin, E.. J. of Quantum Electronics, 132166-2009 (2010).

NOISE INPUTS INTO FEL AMPLIFIER
(NOISE EQUIVALENT POWER - NEP)

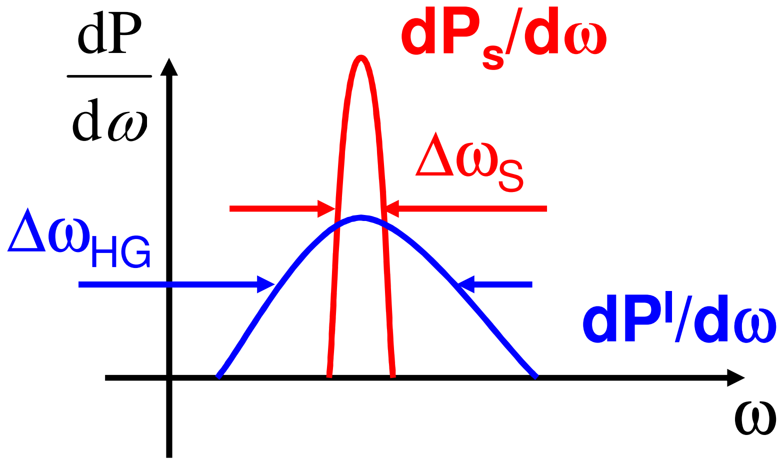


Effective Input noise (NEP)

$$\left(\frac{dP_{in}^{noise}}{d\omega}\right)_{eff} = \left(\frac{dP(L_w)}{d\omega}\right)_{incoh} \Big/ G(\omega)$$

Coherence condition

$$\left[P_s(0)\right]_{coh} >> \left(\frac{dP_{in}^{noise}}{d\omega}\right)_{eff} \Delta\omega$$



To dominate Current Shot-Noise:

$$\left[P_s(0)\right]_{coh} >> \frac{eI_b Z_0}{16\pi A_{em}}\left(\frac{a_w}{\gamma\beta_z\Gamma}\right)^2 \Delta\omega$$ **(seed radiation injection)**

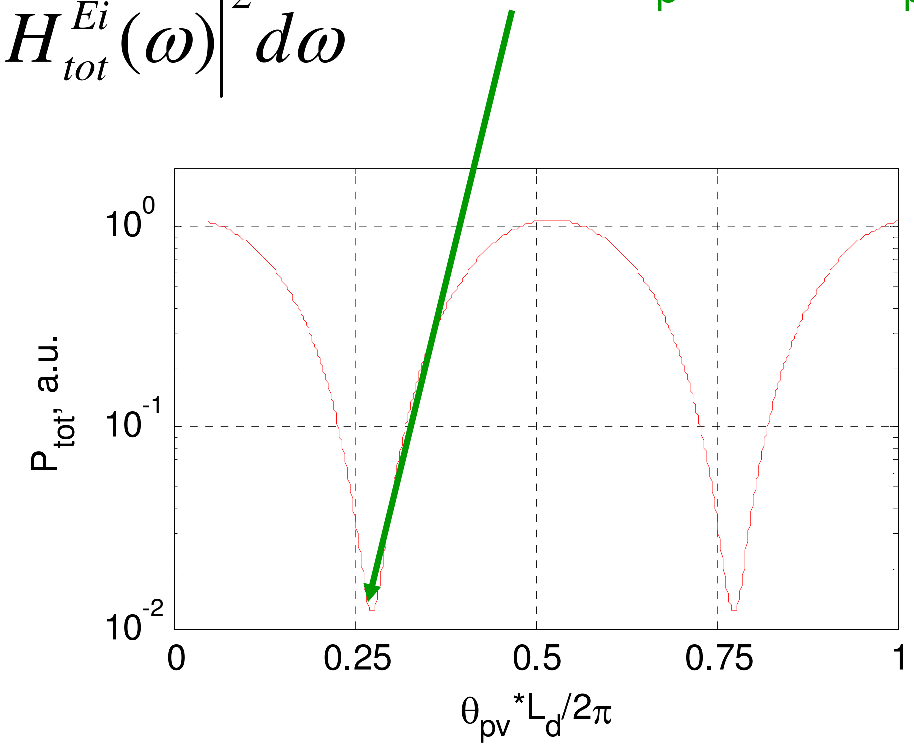
$$\left|\tilde{i}_s(0)\right|^2 \rangle\rangle\ eI_b\Delta\omega$$ **(pre-bunching)**

SASE Power Control

$$P_{tot} = \int \frac{dP}{d\omega} d\omega = \frac{2}{\pi}\left(eI_b\right)\int \left|H_{tot}^{Ei}(\omega)\right|^2 d\omega$$

(starting from current shot-noise at z=0, ΔE=0)

$Ld \sim \lambda_p/4 = \pi/2\theta_p$



E. Dyunin, A. Gover, NIM **A 593**, 49 (2008)

Conditions for **SASE** suppression

Gover, A. Dyunin, E.. J. of Quantum Electronics, 132166-2009 (2010).

Drift + Wiggler

$$\begin{pmatrix} \breve{C}_q(z) \\ \breve{I}(z) \\ \breve{V}(z) \end{pmatrix} = \begin{pmatrix} H^{EE} & H^{EI} & H^{EV} \\ H^{IE} & H^{II} & H^{IV} \\ H^{VE} & H^{VI} & H^{VV} \end{pmatrix} \bullet \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_p & -\frac{i}{W_d}\sin\phi_p \\ 0 & -iW_d\sin\phi_p & \cos\phi_p \end{pmatrix} \bullet \begin{pmatrix} \breve{C}_q(0) \\ \breve{I}(0) \\ \breve{V}(0) \end{pmatrix}$$

$$S = \frac{iW_d H^{EV}}{H^{EI}} = \frac{\gamma_0}{\gamma_z}\frac{\theta_{pw}}{\Gamma}$$

Minimum SASE radiation power when $\phi_p = \frac{\pi}{2} - \frac{\sqrt{3}}{2}S$

Current noise dominance (in SASE): N < S < 1

$$\left(\frac{dP_{in}}{d\omega}\right)^{eff} = \left(\frac{S}{2}\right)^2 \left(\frac{dP_{in}}{d\omega}\right)^{I}_{conv} = \frac{2}{\pi}\frac{\gamma_0^2}{\gamma_z^2}\frac{1}{k\Gamma A_e} eI_b$$

Velocity noise dominance (in SASE): S < N < 1

$$\left(\frac{dP_{in}}{d\omega}\right)^{eff} = \frac{2}{\pi}\frac{\gamma_z^2}{\gamma_0^2} k\Gamma A_e \frac{\left(mc^2\delta\gamma\right)^2}{eI_b}$$

Fundamental “Schawlow-Townes” Coherence Limits (NEP)

e-Beam current noise
+ energy shot
+ radiation noise:

$$\left(\frac{dP_{in}}{d\omega}\right)_{\min} = A \bullet eI_b + B \bullet \cancel{\delta E_c} + \cancel{\frac{\hbar\omega}{1+e^{-\hbar\omega/kT}}}$$

Minimum (energy spread limited) e-beam noise:

$$\left(\frac{dP_{in}}{d\omega}\right)_{\min} = \frac{\delta E_c}{\pi} + \frac{\hbar\omega}{1+e^{-\hbar\omega/kT}}$$

Microwave/THz regime:

$$\left(\frac{dP_{in}}{d\omega}\right)_{\min} = \frac{\delta E_c}{\pi} + \cancel{k_B T} \qquad \left(\approx \frac{\delta E_c}{\pi} = \frac{k_B T_c}{\pi} > k_B T\right)$$

(Cathode temperature limited)

Optical/X-UV regime:

$$\left(\frac{dP_{in}}{d\omega}\right)_{\min} = \cancel{\frac{\delta E_c}{\pi}} + \hbar\omega \qquad (\approx \hbar\omega)$$

(Quantum limit)

Conditions for **CURRENT-NOISE** suppression

Phys. Rev. Lett. 102, 154801 (2009), Phys. Plasmas 18, 123102 (2011)

$$gain = \frac{\overline{\left|\breve{i}(L_d,\omega)\right|^2}}{\overline{\left|\breve{i}(0,\omega)\right|^2}} = \cos^2\phi_p + N^2\sin^2\phi_p$$

$$N^2 = \frac{\overline{\left|\breve{V}(0,\omega)\right|^2}}{W_d^2\left|\breve{i}(0,\omega)\right|^2} = \left(\frac{\lambda_D}{\lambda}\right)^2 \qquad k_D = \frac{2\pi}{\lambda_D} = \frac{\omega_{pl}}{\delta v}$$

$$gain(\phi_p = \pi/2) = N^2$$

For significant suppression (and negligible Landau damping):

$$N = \left(\frac{\lambda_D}{\lambda}\right) << 1$$

Additional conditions:

$$n_0 A_e \lambda >> 1$$

Ballistic condition:

$$\Delta\varphi_b = kL_d\Delta\left(1/\beta_z\right) << 1 \Rightarrow \begin{cases} \dfrac{\Delta\gamma}{\gamma} << \dfrac{\gamma_0^2\lambda}{2L_d} \\ \varepsilon_n << \gamma_0\sigma_{x_0}\left(\dfrac{\lambda}{L_d}\right)^{\frac{1}{2}} \end{cases}$$

SHORT WAVELENGTH LIMITS OF COHERENCE ENHANCEMENT

LCLS parameters: $E = 13GeV \quad I_0 = 3400\,Amp \quad \delta E \sim 0.3MeV \quad \lambda = 1.5A$

For significant suppression: $$N = \left(\frac{\lambda_D}{\lambda}\right) \sim 0.03 << 1$$

Additional conditions: $$n_0 A_e \lambda = \frac{I_0}{ec}\lambda \sim 10^4 >> 1$$

Ballistic condition: $$\Delta\varphi_p = kL_d \Delta\left({}^{1}\!/_{\beta_z}\right) << 1 \Rightarrow \begin{cases} \dfrac{\Delta\gamma}{\gamma} << \dfrac{\gamma_0^2 \lambda}{2L_d} \\ \varepsilon_n << \gamma_0 \sigma_{x0} \left(\dfrac{\lambda}{L_d}\right)^{\frac{1}{2}} \end{cases}$$

Noise suppression factor: $$\left(\frac{S}{2}\right)^2 = \left(\frac{\gamma_0}{\gamma_z}\frac{\theta_{pw}}{\Gamma}\right)^2 << 1$$

E-BEAM NOISE AND RADIATION SUPPRESSION THEORY

Microwave tube noise suppression
H. Haus and F. N. H. Robinson, Proc. IRE 43, 981 (1955). (!)

Optical noise suppression in a drifting relativistic beam:

Gover, Phys. Rev. Lett. 102, 154801 (2009),

Nause, JAP, 107, 103101 (2010)

Optical noise suppression with the dispersive section:

Rathner, PhysRevSTAB 14 060710 (2011)

Gover, Phys. Plasmas 18, 123102 (2011)

SASE noise suppression:

Gover, JQE46, 1511 (2010)

Short wavelength limit:

R. Bonifacio, Optics Communications 138 (1997) 99-100

K-J Kim, FEL conférence 2011: Microbunching workshop UMD (2012)

CONCLUSION

- It is possible to adjust the e-beam current shot-noise level by controlling the longitudinal plasma oscillation dynamics.
- We have demonstrated for the first time such noise suppression at optical frequencies.
- This can be used to enhance FEL coherence and relax seeding power requirement.
- After elimination of shot noise, IR/XUV FEL coherence is ultimately limited by the quantum input noise $dP / d\omega = \hbar\omega$.