The Challenges of a Storage Ring-based Higgs Factory

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Caveat emptor

This is a zero-order pedagogical look based on basic accelerator physics. My numbers are not CERN’s numbers, but they are quite close (≈5%).

For a more precise analysis based on a real lattice design look at arXiv: 1112.2518.pdf by F. Zimmermann and A. Blondel.
Scenario: LHC has discovered the Higgs

- Your HEP friends want to study its properties
  - “Monte Carlo studies show that you need ~ 25 K Higgs for a paper that can get the cover of Nature”
  - They & their students don’t want to be on shift for a lifetime

- They comes to you, his favorite machine builder
  “We need to build a factory to produce 6000 Higgs per year. Projected costs (€ 15 B) all but killed the ILC. Now we know that we don’t need 500 GeV. What about something half that energy?”

- You reply,
  - “You don’t understand about linacs. Half the energy costs you 75% of the original price.”

“Let’s try something different - a storage at CERN. After all LEP 2 got up to 209 GeV.”
What LEP2 might have seen
How can we produce a Higgs with $e^+e^-$?

They respond, “Exactly, but they did not see anything!
The cross-section $\sim 2$ fb. They would have had to run for decades.
A muon collider would be ideal. The $\sigma_H$ is 40,000 times larger.”
“True,” you reply, “be we don’t even know if it is possible.

Let’s go back to storage rings. How much energy do you need?”
Dominant reaction channel with sufficient $\sigma$

- $e^+ + e^- \rightarrow Z^* \rightarrow H + Z$

- $M_H + M_Z = 125 + 91.2 = 216.2 \text{ GeV/c}^2$

$\Rightarrow$ set our CM energy at the peak $\sigma$: $\sim 240 \text{ GeV}$
Physics “facts of life” of a Higgs factory
Will this fit in the LHC tunnel?

- Higgs production cross section \( \sim 220 \text{ fb} \ (2.2 \times 10^{-37} \text{ cm}^2) \)

- Peak \( \mathcal{L} = 10^{34} \text{ cm}^{-1} \text{ s}^{-1} \implies \langle \mathcal{L} \rangle \sim 10^{33} \text{ cm}^{-1} \text{ s}^{-1} \)

- \(~30 \text{ fb}^{-1} / \text{ year} \implies 6600 \text{ Higgs} / \text{ year}~

- Total \( e^+e^- \) cross-section is \( \sim 100 \text{ pb} \cdot (100 \text{GeV/E})^2 \)
  - Will set the luminosity lifetime

We don’t have any choice about these numbers

Oh, and don’t use more than 200 MW of electricity
Road map for the analysis

- How do “facts of life” affect the peak luminosity
  - First some physics about beam-beam interactions
    ==> Luminosity as function of $I_{\text{beam}}$ and $E_{\text{beam}}$
  - What $\beta^*$ is needed?

- What is the bunch length, $\sigma_z$, of the beam?

- How does rf system give us $\sigma_z$
  - What are relevant machine parameters, $\alpha_c$, $f_{\text{rev}}$, $f_{\text{rf}}$, $\phi_{\text{synch}}$, etc.
  - But first, what is $\Delta E/E$

- How synchrotron radiation comes in
  - What is the rf system
  - What sets the beam size at the IP

- What are life time limitations

- Conclusions
Storage ring physics: Beam-beam tune shifts
Space charge fields at the Interaction Point

At the IP space charge cancels; but the currents add ==> the IP is a “lens”
i.e., it adds a gradient error to the lattice, (k_{space charge} \Delta s)
where (k_{space charge} \Delta s) is the kick (“spring constant”) of the space charge force

Therefore the tune shift is

\[ \Delta Q = -\frac{1}{4\pi} \beta^*(s)(k\Delta s) \]

For a Gaussian beam, the space charge kick gives

\[ \Delta Q \approx \frac{r_e}{2} \frac{\beta^* N}{\gamma A_{int}} \]
Effect of tune shift on luminosity

- The luminosity is
  \[ \mathcal{L} = \frac{f_{\text{coll}} N_1 N_2}{4 A_{\text{int}}} \]

- Write the area in terms of emittance & $\beta$ at the IR ($\beta^*$)
  \[ A_{\text{int}} = \sigma_x \sigma_y = \sqrt{\beta_x^* \varepsilon_x} \cdot \sqrt{\beta_y^* \varepsilon_y} \]

- For simplicity assume that
  \[ \frac{\beta_x^*}{\beta_y^*} = \frac{\varepsilon_x}{\varepsilon_y} \Rightarrow \beta_x^* = \frac{\varepsilon_x}{\varepsilon_y} \beta_y^* \Rightarrow \beta_x^* \varepsilon_x = \frac{\varepsilon_x^2}{\varepsilon_y} \beta_y^* \]

- In that case
  \[ A_{\text{int}} = \varepsilon_x \beta_y^* \]

- And
  \[ \mathcal{L} = \frac{f_{\text{coll}} N_1 N_2}{4 \varepsilon_x \beta_y^*} \sim \frac{I_{\text{beam}}^2}{\varepsilon_x \beta_y^*} \]
To maximize luminosity, increase \( N \) to the tune shift limit

- We saw that
  \[
  \Delta Q_y \approx \frac{r_e \beta^* N}{2 \gamma A_{\text{int}}}
  \]

Or, writing \( N \) in terms of the tune shift,

\[
N = \Delta Q_y \frac{2\gamma A_{\text{int}}}{r_e \beta^*} = \Delta Q_y \frac{2\gamma \epsilon_x \beta^*}{r_e \beta^*} = \frac{2}{r_e} \gamma \epsilon_x \Delta Q_y
\]

Therefore the tune shift limited luminosity is

\[
\mathcal{L} = \frac{2}{r_e} \Delta Q_y \frac{f_{\text{coll}} N_1 \gamma \epsilon_x}{4 \epsilon_x \beta_y^*} \sim \Delta Q_y \left( \frac{IE}{\beta_y^*} \right)
\]
Tune shift limited luminosity of the collider

\[ L = \frac{N^2 c \gamma}{4\pi \varepsilon_n \beta^* S_B} = \frac{1}{er_i m_i c^2} \frac{N r_i}{4\pi \varepsilon_n} \left( \frac{E I}{\beta^*} \right) = \frac{1}{er_i m_i c^2} \frac{N r_i}{4\pi \varepsilon_n} \left( \frac{P_{\text{beam}}}{\beta^*} \right) \]

\( i = e, p \)

Linear or Circular

Tune shift

In practical units for electrons

\[ \mathcal{L}_{\text{peak}} = 2.17 \cdot 10^{34} \left( 1 + \frac{\sigma_x}{\sigma_y} \right) \Delta Q_y \left( \frac{1 \text{ cm}}{\beta^*} \right) \left( \frac{E}{1 \text{ GeV}} \right) \left( \frac{I}{1 \text{ A}} \right) \]

Experimentally, at the tune shift limit \( \left( 1 + \frac{\sigma_x}{\sigma_y} \right) \Delta Q_y \approx 0.1 \) for electrons

\[ \mathcal{L}_{\text{peak}} = 2.17 \cdot 10^{33} \left( \frac{1 \text{ cm}}{\beta^*} \right) \left( \frac{E}{1 \text{ GeV}} \right) \left( \frac{I}{1 \text{ A}} \right) \]
We can only choose $I(A)$ and $\beta^*(cm)$

- For the LHC tunnel with $f_{\text{dipole}} \sim 2/3$, $\rho_{\text{curvature}} \sim 2700$ m
- Remember that
  \[
  \rho(m) = 3.34 \left( \frac{p}{1 \text{ GeV/c}} \right) \left( \frac{1}{q} \right) \left( \frac{1 \text{ T}}{B} \right)
  \]
- Therefore, $B_{\text{max}} = 0.15$ T
- Per turn, each beam particle loses to synchrotron radiation
  \[
  U_o(\text{keV}) = 88.46 \frac{E^4(\text{GeV})}{\rho(m)}
  \]
  or 6.54 GeV per turn

$I_{\text{beam}} = 7.5$ mA $\implies \sim 100$ MW of radiation (2 beams)
CERN management “chose” I;
That leaves $\beta^*$ as the only free variable

- Then
  
  $$L_{\text{peak}} \approx 1.9 \times 10^{33} \left(\frac{1 \text{ cm}}{\beta^*}\right)$$

- Therefore to meet the luminosity goal
  
  $$<\beta^*_x \beta^*_y>^{1/2} \sim 0.2 \text{ cm} \quad (10 \times \text{smaller than LEP 2})$$

- Is this possible? Recall that is the depth of focus at the IP

  The “hourglass effect” lowers $L$

  For maximum luminosity

  $$\Rightarrow \sigma_z \sim \beta^* \sim 0.2 \text{ cm}$$
The analysis of longitudinal dynamics gives

\[ \sigma_z = \frac{c \alpha_c \sigma_p}{\Omega_{sync} p_0} = \left( \frac{c^3}{2 \pi q} \right) \left( \frac{p_0 \beta_0 \eta_c}{h f_0^2 \hat{V} \cos(\varphi_s)} \right) \frac{\sigma_p}{p_0} \]

where \( \alpha_c = (\Delta L/L) / (\Delta p/p) \)

If the beam size is \(~100\ \mu m\) in most of the ring

\[ \frac{\Delta L}{L} < \frac{0.01}{280000} \approx 3 \times 10^{-7} \]

for electrons to stay within \( \sigma_x \) of the design orbit

To know bunch length & \( \alpha_c \) we need to know \( \Delta p/p \sim \Delta E/E \)
Bunch length, \( \sigma_z \), is determined by \( \Delta E/E \)

- For electrons to a good approximation
  \[
  \Delta E \approx \sqrt{E_{\text{beam}}} < E_{\text{critical, photon}} >
  \]
  and
  \[
  \epsilon_c[keV] = 2.218 \frac{E[GeV]^3}{\rho[m]} = 0.665 \cdot E[GeV]^2 \cdot B[T]
  \]

- So \( \epsilon_{\text{crit}} \approx 1.5 \text{ MeV} \) \( \implies \Delta E/E \approx 0.0035 \)

- Therefore for electrons to remain near the design orbit
  \[
  \alpha_c = (\Delta L/L) / (\Delta p/p) \sim 8 \times 10^{-5}
  \]
  \( \text{(was } 1.8 \times 10^{-4} \text{ for LEP2)} \)
The rf-bucket contains $\Delta E/E$ in the beam

- As $U_o \sim 6.5$ GeV,
  
  $V_{rf,max} > 6.5$ GeV + “safety margin” to contain $\Delta E/E$

- Some addition analysis

  \[
  \left( \frac{\Delta E}{E} \right)_{\text{max}} = \sqrt{\frac{q\hat{V}_{\text{max}}}{\pi h \alpha_c E_{\text{synchronous}}}} \left( 2\cos\varphi_s + (2\varphi_s - \pi)\sin\varphi_s \right)
  \]

  where $h$ is the harmonic number ($\sim C_{\text{LEP3}} / \lambda_{rf} \sim 9 \times 10^4$)

- The greater the over-voltage, the shorter the bunch

  \[
  \sigma_s = \frac{c\alpha_c}{\Omega_{\text{synch}}} \left( \frac{\Delta E}{E} \right) = \sqrt{\frac{c^3}{2\pi q} \frac{p_0\beta_0 \alpha_c}{h f_{rev}^2 \hat{V}_{\text{max}} \cos(\varphi_s)}} \left( \frac{\Delta E}{E} \right)
  \]
For the Higgs factory…

- The maximum accelerating voltage must exceed 9 GeV
  - Also yields $\sigma_z = 3$ mm which is okay for $\beta^* = 1$ mm

- A more comfortable choice is 11 GeV (it’s only money)
  - $\Rightarrow$ CW superconducting linac for LEP 3 $\Rightarrow \phi_{synch}$

- Therefore, we need a SCRF linac in 4 pieces
  - Remember that the beam loses $\sim 6\%$ of its energy in one turn
    - LEP2 lost 3.4 GeV $\sim 3\%$ per turn
  - We need a higher gradient than LEP2; 6 MeV/m is not enough
    - 22 MeV/m $\Rightarrow$ 500 m of linac (the same as LEP 2)

- High gradient $\Rightarrow f_{rf} > 1$GHz ;
For the Higgs factory...

- The maximum accelerating voltage must exceed 9 GeV
  - Also yields $\sigma_z = 3$ mm which is okay for $\beta^* = 1$ mm

- A more comfortable choice is 11 GeV (it’s only money)
  - $\Rightarrow$ CW superconducting linac for LEP 3
  - This sets the synchronous phase

- For the next step we need to know the beam size

  $$\sigma_i^* = \sqrt{\beta_i^* \varepsilon_i} \text{ for } i = x, y$$

- Therefore, we must estimate the natural emittance which is determined by the synchrotron radiation $\Delta E/E$
The minimum horizontal emittance for an achromatic transport

\[ \varepsilon_{x,\text{min}} = 3.84 \times 10^{-13} \left( \frac{\gamma^2}{J_x} \right) F_{\text{min}} \text{ meters} \]

\approx 3.84 \times 10^{-13} \gamma^2 \left( \frac{\theta_{\text{achromat}}^3}{4\sqrt{15}} \right) \text{ meters} \]

\[ \varepsilon_y \sim 0.01 \varepsilon_x \]
Because $\alpha_c$ is so small, we cannot achieve the minimum emittance.

- For estimation purposes we will choose $20 \varepsilon_{\text{min}}$ as the mean of the x & y emittances.

- For the LHC tunnel a maximum practical dipole length is 15 m.
  - A triple bend achromat $\sim$ 80 meters long $\Rightarrow \theta = 2.67 \times 10^{-2}$

\[
<\varepsilon> \sim 7.6 \text{ nm-rad} \Rightarrow \sigma_{\text{transverse}} = 2.8 \mu\text{m}
\]

*How many particles are in the bunch?*

*Or how many bunches are in the ring?*
We already assumed that the luminosity is at the tune-shift limit

We have

\[
L = \frac{N^2 c \gamma}{4 \pi \varepsilon_n \beta^* S_B} = \frac{1}{er_i m_i c^2} \left( \frac{N_{i_r}}{4 \pi \varepsilon_n} \right) \left( \frac{EI}{\beta^*} \right) = \frac{1}{er_i m_i c^2} \frac{N_{i_r}}{4 \pi \varepsilon_n} \left( \frac{P_{\text{beam}}}{\beta^*} \right) \quad i = e, p
\]

Or

\[Q = \frac{N_{i_r}}{4 \pi \varepsilon \gamma} \quad \Rightarrow \quad N = \frac{4 \pi \varepsilon \gamma}{r_e} Q\]

So,

\[N_e \sim 1.3 \times 10^{11} \text{ per bunch}\]

\[I_{\text{beam}} = 7.5 \text{ mA} \implies \text{there are only 3 bunches in the ring}\]
Let’s return to Space charge fields at the collision point

At the IP space charge cancels; currents add

=> strong beam-beam focus
  => Luminosity enhancement
  => Very strong synchrotron radiation (beamstrahlung)

Beamstrahlung is important in linear colliders

What about the beams in LEP-3?
At the collision point…with $\mathcal{L} = 10^{34}$

$$I_{\text{peak}} = \frac{N_e}{2} \sigma_z \implies I_{\text{peak}} \sim 1.6 \text{ kA}$$

- Therefore, at the beam edge ($\sigma$)

$$B = I(A)/5r(\text{cm}) = 1.6 \text{ MG}!$$

- When the beams collide they emit synchrotron radiation (beamstrahlung)

$$\varepsilon_{c,\text{Beams}}[\text{keV}] = \frac{2.218 E[\text{GeV}]^3}{\rho[\text{m}]} = 0.665 \cdot E[\text{GeV}]^2 \cdot B[T] = 1.1 \text{ GeV}$$

- But this accumulates over a damping time

$$\Delta E_{\text{Beams}} \approx (2/J_E) \cdot \text{Sqrt (number of turns in damping time)} \cdot \varepsilon_{c,\text{Beams}} \approx 10 \text{ GeV}$$

*The rf-bucket must be very large to contain such a big $\Delta E/E$*

*Beamstrahlung limits beam lifetime & energy resolution of events*
At $\mathcal{L} = 2 \times 10^{33}$

- $\beta^* \sim 1.5$ cm $\implies$ 9 GeV of linac is okay

- $I_{\text{peak}}$ can be reduced 3 x and …

- The beam size can increase 3 x

$\implies$ $B_{\text{sc}}$ is reduced $\sim 10$ x $\implies$ $\Delta E_{\text{Beams}} \sim 1$ GeV
  - This is < 1% of the nominal energy
  - Many fewer electrons will be lost

*A much easier machine to build and operate*
Yokoya has done a more careful analysis

- Beamstrahlung limited luminosity

\[ \mathcal{L} = 4.57 \times 10^{33} \left( \frac{\rho}{1 \text{ km}} \right) \left( \frac{P_{SR}}{100 \text{ MW}} \right) \sqrt{\frac{\Delta E_{\text{beams}}}{E}} \left( \frac{100 \text{ GeV}}{E} \right)^4 \left( \frac{1 \text{ nm}}{\varepsilon_y} \right)^{1/2} \text{ cm}^{-2} \text{s}^{-1} \]

- This implies very large rings, high beam power, and small vertical emittance
Mechanisms limiting beam lifetime

- Luminosity lifetime
  
  \[ \text{Total } e^+e^- \text{ cross-section is } \sim 100 \text{ pb } \cdot (100\text{GeV/E})^2 \]

- Beamstrahlung lifetime

- Beam-gas scattering & bremsstrahlung

- Touscheck lifetime

- And…
And there are other problems

- Remember the Compton scattering of photons up shifts the energy by $4 \gamma^2$

  $$E = \gamma mc^2$$

- Where are the photons?
  - The beam tube is filled with thermal photons (25 meV)

- In LEP-3 these photons can be up-shifted as much as 2.4 GeV
  - 2% of beam energy cannot be contained easily
  - We need to put in the Compton cross-section and photon density to find out how rapidly beam is lost
The bottom line: The beam lifetime is 10 minutes

- We need a powerful injector
- Implies rapid decay of luminosity as operation shrinks away from tune shift limit

\[ \Rightarrow \text{we need top-off operation} \]
Conclusions (for $L = 2 \times 10^{34}$)

- LEP3 is a machine at the edge of physics feasibility
  - Beamstrahlung issues require more, detailed study
  - Momentum aperture must be very large
  - 240 GeV is the limit in the LHC tunnel

- The cost appears to be $\ll$ a comparable linear collider
- A very big perturbation of LHC operations
- Cannot run at the same time as the LHC

The LEP3 idea might be a viable alternative as a future HEP project